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ABSTRACT

This manual was designed for use by teachers using the School Mathematics Study Group's (SMSG) special text series for low achievers in grades 7 and 8; it covers chapters 10 through 18 of that series. The manual begins with introductory material describing characteristics of low-achieving students and suggested instructional approaches. Testing policies, classroom routine, and necessary materials and supplies are also discussed. For each chapter of the text this volume lists and describes objectives, suggests special approaches where desirable, and provides solutions to all problems posed in the student text. (SD)

SECONDARY SCHOOL MATHEMATICS

SPECIAL EDITION

TEACHER'S COMMENTARY

Chapters 10-18

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PREFACE

Introduction

This is a program written explicitly for low-achieving mathematics students at the 7th and 8th grade levels. The material closely parallels the new SMSG Secondary School Mathematics project. The same major topics found in the regular edition are presented, but without the depth and the verbalization of structural ideas.

The program is based on an earlier project the results of which are reported in SMSG Reports Numbers 6 and 7. It has been tried with low-achieving junior high school students including classes of students from minority groups .

Characteristics of Low-Achieving Students

Observation of typical students who fall into the category of low-achieving math students leads to certain assumptions regarding the pre-conditioning these students have experienced.

First, it is likely that they have a poor image of themselves as students in general and of mathematics in particular, partly as a result of a history of failure. It is essential that the student's self-image improve if he is to learn. The text material attempts to provide built-in success through the use of developmental class discussion exercises and pre-tests. For effective use, the teacher must exhaust every opportunity to ensure the student's success. The pre-tests give the student the opportunity to make high scores on the tests. The teacher is urged to reward the student with appropriately high grades whenever possible.

Second, these students may be preoccupied with their own emotional or physical problems, and these may very well contribute to their disorganization. Additional organizational demands such as having books, pencils, homework, etc., are often unrealistic expectations. Many times the best that can be hoped for is that the student gets to class more or less on time. The teacher should be prepared to supply students with pencils as needed without penalty.

In order to minimize the demands on the student, a lesson a day is handed out to be filed in their notebooks and kept in class. All work is to be done in class. This relieves the student of the burden of remembering his text and/or homework and assures him of a source of help when he needs it. Each page of a lesson is numbered with the same number followed by a letter. For example, Chapter 1, Lesson 1, consists of pages 1-1, 1-1a, 1-1b. Lesson 2 is numbered 1-2, 1-2a, 1-2b, 1-2c, 1-2d, and 1-2e. All the pages of a lesson are to be handed out at the same time.

Third, it is probable that these students have been denied the opportunity to attempt more conceptual mathematics because of a lack of computational skills. Typically, they have been subjected to a rote drill approach in an effort to strengthen these skills. Often this has the effect of increasingly boring the student and reinforcing his negative attitude toward mathematics. An effort has been made to relieve the student of much computation through extensive use of tables. It is felt that this will allow the student to focus upon learning concepts and that this will be a more interesting and rewarding

approach. There is some indication that through repeated use of tables the student does, in fact, improve his computation skills without the routine of excessive drill.

Fourth, these students characteristically exhibit a short attention span and a short interest span. In some cases this is a reflection of poor reading skills. This problem is dealt with by the format of each lesson. Typically, there is a short introduction followed by a developmental class discussion exercise. These exercises must be discussed with as much student involvement as possible. Rejection of a student's incorrect response only reinforces his unwillingness to involve himself in mathematics. Lecture methods of presentation are definitely not recommended, as this method provides the student with a built-in mental escape. Even though it is difficult to achieve, student involvement is the key.

Class discussion is followed by an exercise set. The teacher is cautioned not to "abandon" the class at this time. Rather, the teacher should circulate about the class making available all individual help necessary. Having correct answers in the book is regarded as evidence of success and a source of pride. The teacher is urged to capitalize on this by ensuring that the student gets correct answers even though initially the teacher may have to supply the answer. As the student's self-assurance improves, he becomes more willing to risk answers of his own. Even then he may need frequent reassurance. "Is this right?" is an often repeated question. Always give the student all the encouragement he needs.

Every effort has been made to keep the reading short and at realistic levels. Even so the teacher should be prepared to read material to certain students.

Tests

At the end of each chapter there are three tests. The first, a pre-test, is intended as a teaching device. Each item is referenced to a section number in the chapter just completed. Answers are found in this commentary. The student is instructed to re-read the sections corresponding to incorrect items. If he still needs assistance he is further instructed to "ask his teacher". Students soon learn that a good understanding of the pre-test items is almost a guarantee of success on the test following. Some pre-tests are lengthy and may take more than one period.

The chapter test following the pre-test is intended for student evaluation. In most cases it is identical to or closely resembles the pre-test. This is intentional as it provides the student with the opportunity to receive a creditable grade as a tangible reward for his efforts.

The student should be permitted to use his tables for all work including the pre-tests and chapter tests.

After the chapter test there is a short review test of selected previous topics. Answers are given following this "Check Your Memory: Self-Test". The student should grade himself. No additional review is recommended.

Because sections from earlier chapters are reviewed, the teacher should make sure that when each chapter is finished the text is included in the classroom library. For instance, students working in Chapter 13 should have the complete texts of Chapters 1 through 12 readily available.

Classroom Routine

These students show evidence that they are more comfortable if a set classroom routine is followed daily. A suggested routine is as follows.

1. Pass out student notebooks.
2. Pass out the daily lesson.
3. Supply pencils as needed.
4. Begin the lesson.
5. Take as much time as required on Class Discussion.
6. Have students do the exercises. Make yourself available for individual help during this time.
7. Collect notebooks which are retained in the classroom.

Materials and Supplies

1. The material should be reproduced on three-hole punched paper for easy filing in student notebooks.
2. A three-ring looseleaf binder is needed for each student. About 5 chapters will fill a three-ring binder at which time the student should transfer the material to a file which is accessible for ready reference.
3. A classroom set of compasses
4. A classroom set of scissors
5. A classroom set of protractors
6. A classroom set of 6" straight-edges. These can be made easily and cheaply by cutting up pieces of tag board or heavy manila paper such as file folders and can be considered expendable.

Teacher's Commentary

TABLE OF CONTENTS

Page

Teachers Commentary

Chapter 10

DECIMALS

Some students who have trouble computing with fractions find it easier to use decimal equivalents. To do this, however, they need to understand both place value and the relationship between fractions and decimals.

Because all students have had some experience in computation involving money, the decimal point is introduced in terms of dollars and cents.

The objectives of this chapter are:

1. to teach the short division algorithm;
2. to extend the concept of place value to include numbers less than 1 ;
3. to teach students how to rename decimals as fractions and vice versa;
4. to teach students to add, subtract, multiply, and divide using decimals.

Throughout the chapter, the emphasis is on the fact that the decimal point separates whole numbers from numbers less than 1 . In the summary at the end of the chapter, the student is urged to find out whether an answer is reasonable by looking at the whole numbers involved. The <u>teacher</u> should introduce this in discussion as soon as students begin operations on decimal numbers to prevent mistakes like these: 5.6×3.1 equals 173.6 , or $2.059 + 7.1$ equals 21.30 or 2.130 , or 213.0 .

Most important of all, we want students to <u>believe</u> that a fraction and its decimal equivalent really name the same number.

Lesson 10-1.

The short division algorithm speeds up division with one-digit divisors, which are the ones most often used in renaming fractions as decimals. Some students may be reluctant to give up the long division algorithm and of course should not be forced to do so. Most, however, after a little practice find division less tedious and make fewer mistakes when they use the shorter algorithm.

Class Discussion - Page 10-1c.

1. (a) 400
   (b) 2
   (c) 200
   (d) 86
   (e) 4
   (f) 40
   (g) 6
   (h) 3
   (i) 0
   (j) $\frac{486}{2} = \underline{243}$

2. (a) 300
   (b) No
   (c) Yes
   (d) 200
   (e) 100
   (f) 184
   (g) 180
   (h) 9
   (i) 90
   (j) 4
   (k) 2
   (ℓ) $\frac{384}{2} = \underline{192}$

3. (a) No
   (b) No
   (c) 35
   (d) $7 \times \underline{5} = 35$ so $7 \times \underline{500} = 3500$
   (e) 1 hundred is left
   (f) No

(g) 7

(h) $7 \times \underline{1} = 7$ so $7 \times \underline{10} = 70$

(i) $13 - 7 = \underline{6}$ so $130 - 70 = \underline{60}$

(j) $7 \times \underline{9} = 63$

(k) $\frac{3633}{7} = \underline{519}$

Exercises - Page 10-1g.

1. 121
2. 212
3. $166 \frac{1}{3}$
4. 162
5. $813 \frac{4}{6}$ (or $813 \frac{2}{3}$)
6. 862
7. $821 \frac{2}{6}$ (or $821 \frac{1}{3}$)
8. $724 \frac{8}{9}$
9. $991 \frac{4}{8}$ (or $991 \frac{1}{2}$)
10. 772
11. $557 \frac{2}{9}$
12. 873

Lesson 10-2.

Class Discussion - Page 10-2.

1. (a) 1000

   (b) No

   (c) No

   (d) No

   (e) 150

   (f) $25 \times \underline{6} = 150$ so $25 \times \underline{60} = 1500$

   (g) 7

   (h) $\frac{1675}{25} = \underline{67}$

2. (a) $21 \times \underline{700} = 14700$

The 7 in the answer means $\underline{700}$.

(b) $21 \times \underline{10} = 210$

The 1 in the answer means $\underline{10}$.

(c) $21 \times \underline{3} = 63$

The 3 in the answer means $\underline{3}$.

(d) No. $\frac{18}{21} = \frac{6}{7}$

(e) $\frac{14991}{21} = \underline{713 \frac{6}{7}}$

Exercises - Page 10-2b.

1. 137
2. $1210 \frac{3}{8}$
3. $6636 \frac{3}{4}$
4. $1140 \frac{3}{7}$
5. $54 \frac{8}{30}$ (or $54 \frac{4}{15}$)
6. $20 \frac{34}{44}$ (or $20 \frac{17}{22}$)

Lesson 10-3.

Exercises - Page 10-3b.

2. \$1.55
3. \$1.60
4. \$49.35
5. \$.95
6. \$8.90
7. \$13.75
8. \$.30
9. \$.65
10. \$.05

Lesson 10-4.

Class Discussion - Page 10-4.

63 means 6 tens and 3 ones.

630 means 6 hundreds, 3 tens, and 0 ones.

The digit 6 in 630 has 10 times the value of the 6 in 63.

The digit 3 in 630 has 10 times the value of the 3 in 63.

In money, \$63.00 has 10 times the value of \$6.30. The number of whole dollars in \$63.00 is 63 because that is the number to the left of the decimal point.

With \$6.30, however, we have only 6 whole dollars and $\frac{3}{10}$ of another dollar. If we write \$.63, the 6 means $\frac{6}{10}$ of a dollar and the 3 means $\frac{3}{100}$ of a dollar.

The decimal point separated the whole number of dollars from the part of a dollar less than 1.

In the "times 10" machine, the input is \$1.35 and the output is \$13.50. The decimal point moved one place to the right.

$f: x \longrightarrow 10x$	
Input (dollars)	Output (dollars)
1.35	13.50
2.18	21.80
.57	5.70
4.75	47.50
3.14	31.40

Exercises - Page 10-4b.

With money, when you multiply by 100 the decimal point moves 2 places to the right.

f : x ⟶ 100x	
Input (dollars)	Output (dollars)
1.35	135.00
2.18	218.00
.57	57.00
4.75	475.00
10.00	1000.00
3.14	314.00
.49	49.00
16.83	1683.00
.02	2.00

Lesson 10-5.

Be sure that students understand that zeros are unimportant except when they appear between some non-zero digit and the decimal point.

Class Discussion - Page 10-5a.

To get from any of these numbers to the one underneath it, we move the decimal point one place to the right.

This is multiplying by 10 .

If we start at 1000 and go to 100 ; the decimal point moves one place to the left.

This is dividing 1000 by 10 .

Divide 100 by 10 . 10

To divide 100 by 10 we move the decimal point <u>1</u> place to the <u>left</u>.

If we divide 10 by 10 we get <u>1</u>.

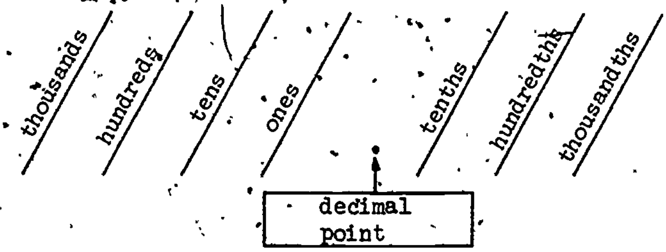
.1 and $\frac{1}{10}$ are the same number.

The second place to the right of the decimal point is <u>hundredths</u> place.

The third place to the right of the decimal point is <u>thousandths</u> place, because .001 and $\frac{1}{1000}$ are the same number.

Exercises - Page 10-5b.

1.



2. (a) 3 4 5. 1 3
   (b) 7 3 0. 4
   (c) 8 4 7 6 0. 9
   (d) 3 7 1 1. 6 6 1
   (e) 6 0. 4 3 4
   (f) 7 5 8 6. 0 2
   (g) 2 8 7. 6 5 9
   (h) 1 3 4 9 5.
   (i) 2 3 7. 9 0 1
   (j) 4 5 2 6. 7 3
   (k) 8 4 7. 5 1
   (ℓ) 1 0 0 8. 9 5 1 6
   (m) 8 9 8. 4 2 3
   (n) 6. 8 3 9 0 5 2

Lesson 10-6.

We ask students to "tack on" enough zeros so that the numbers to be compared have the same number of decimal places. This helps to overcome the tendency simply to write numbers with the last digits lined up, no matter what the place value of the digit. "Tacking on" zeros is brought in again in Section 10-13, Addition and Subtraction with Decimals.

Students generally find it hard to read decimal numerals aloud. Although it is, of course, essential that they know the value of each place, they should not be forced to read 459.7631 as "four hundred fifty-nine and seven thousand six hundred thirty-one ten-thousandths". On the contrary, the separation of whole number from fraction is emphasized when students read the whole number in the usual way, "four hundred fifty-nine" and then say, "point seven six three one".

Class Discussion - Page 10-6a.

1. (a) 3

   (b) 4

   (c) No

2. (a) Yes

   (b) Yes

   (c) No

   (d) $3 > 2$

   (e) $.123 > .1229$

Exercises - Page 10-6c.

1. .07 $>$ .0096
2. .5001 $>$ .0999
3. .0001 $<$ .0091
4. .0286 $>$ .00989
5. .397 $>$ .0309
6. .00001 $>$ .000001
7. 2.03 $<$ 2.079
8. 43.07 $>$ 8.065
9. 593.76 $<$ 593.761
10. 7.605 $>$ 7.6005

Lesson 10-7.

The flow chart for writing decimals as fractions makes for a purely mechanical process, easily remembered, and calls attention to the relationship between the number of zeros in the power of 10 in the denominator and the number of decimal places. This relationship is used in later sections of this chapter and in the presentation of scientific notation in Chapter 13.

Exercises - Page 10-7a.

1. $\frac{9}{100}$
2. $\frac{11}{10}$
3. $\frac{47}{1000}$
4. $\frac{4581}{100}$
5. $\frac{763}{1000}$
6. $\frac{4019}{1000}$
7. $\frac{53}{100}$
8. $\frac{6057}{10000}$
9. $\frac{9}{100000}$
10. $\frac{1000007}{10000}$

Lesson 10-8.

Class Discussion - Page 10-8.

Divide 2000.0 by 5 . You get <u>400</u> . (or 400.0)

$\frac{2000.0}{5}$ and 400.0 are different names for the same number.

Divide 200.0 by 5 . You get <u>40</u> . (or 40.0)

$\frac{200.0}{5}$ and 40.0 are different names for the same number.

Divide 20.0 by 5 . You get <u>4</u> . (or 4.0)

$\frac{20.0}{5}$ and 4.0 are different names for the same number.

The decimal point in the numerator moved one place to the left.

The decimal point in the numbers on the right moved one place to the left.

2.0 divided by 5 is .4 .

$$\frac{2.0}{5} = \frac{2}{5} = .4$$

$$\frac{300.0}{6} = 50 \text{ (or } 50.0)$$

$$\frac{30.0}{6} = 5 \text{ (or } 5.0)$$

$$\frac{3}{6} = .5$$

Exercises - Page 10-8b.

1. (a) $\frac{40}{5} = 8$

   (b) $\frac{4}{5} = .8$

2. (a) $\frac{300}{4} = 75$

   (b) $\frac{30}{4} = 7.5$

   (c) $\frac{3}{4} = .75$

3. (a) $\frac{10}{2} = 5$

   (b) $\frac{1}{2} = .5$

4. (a) $\frac{100}{4} = 25$

   (b) $\frac{10}{4} = 2.5$

   (c) $\frac{1}{4} = .25$

Lesson 10-9.

This section again emphasizes the idea that fractions are division expressions and reinforces the equality or same-ness of .8 and $\frac{4}{5}$, .75 and $\frac{3}{4}$, etc. Repeating decimals are delayed until the next section.

Class Discussion - Page 10-9a.

If we want to change $\frac{13}{10}$ to a decimal, we copy the numerator, 13. There is one zero in the denominator. There will be one decimal place. $\frac{13}{10} = 1.3$

If we want to change $\frac{19}{1000}$ to a decimal, we copy the numerator, 19. There are 3 zeros in the denominator. There will be 3 decimal places.

$\frac{13}{10}$ means 13 divided by 10.

$10\overline{)13}$

Can you divide? Yes.

The left-over 3 is 30 tenths. You can divide. 30 tenths divided by 10 is 3 tenths.

$\frac{13}{10} = 1.3$

$1000\overline{)19}$

Can you divide? No.

19 = 190 tenths

Can you divide? No.

19 = 1900 hundredths

Can you divide? Yes.

Exercises - Page 10-9e.

1. (a) .25
   (b) .375
   (c) .4
   (d) .9
   (e) .875
   (f) .8
   (g) .625
   (h) .3

2. (a) 1.25
   (b) 2.5
   (c) 1.4
   (d) 2.25
   (e) .2
   (f) .15
   (g) .16
   (h) .95
   (i) 1.8
   (j) 1.125

Lesson 10-10.

No attempt is made in the entire chapter to show students how to rename repeating decimals as fractions, but you may encourage them to memorize commonly-used fractions such as $\frac{1}{3} = .\overline{3}$, $\frac{1}{6} = .1\overline{6}$ and $\frac{2}{3} = .\overline{6}$.

Review briefly what prime factorization means. Students do not need to write prime factorizations of denominators but do show them that there is a reason why repeating decimals repeat.

Class Discussion - Page 10-10.

When you divide 10 tenths by 3 , you get 3 with a remainder of 1 tenth.

When you divide 10 hundredths by 3 , you get 3 with a remainder of 1 hundredth.

When you divide 10 thousandths by 3 , you get 3 with a remainder of 1 thousandth.

Since you keep on dividing 10 of something by 3 , you will always get <u>3</u> with a remainder of 1 . The digit <u>3</u> in the decimal will be repeated.

$11\overline{)1.00}$

Can you divide? <u>No</u>.

10 tenths = 100 hundredths

$$\begin{array}{r} .0 \\ 11\overline{)1.00} \end{array}$$

Can you divide? <u>Yes</u>.

(With $\frac{1}{11}$) No matter how long you work you will always get first a 0 , then a <u>9</u> .

$\frac{3}{7} = .\overline{428571}$

Exercises - Page 10-10b.

1. 2.5
2. $.\overline{6}$
3. $.\overline{4}$
4. 1.75
5. $.875$
6. $.8\overline{3}$
7. $.08\overline{3}$
8. $.1\overline{6}$
9. $.125$
10. $.\overline{8}$

Lesson 10-11.

The brief review of algorithms for whole number multiplication reinforces the idea that decimals are multiplied as if they were whole numbers, ~~and then~~ the number of decimal places in the product is calculated. This ties in with the way multiplication of decimals is introduced--through fractions.

Allow students to use any algorithm for multiplication that produces correct answers. In a problem such as 327 × 195, show why the zeros are written in the partial products:

```
  327
× 195
 1635
29430 ← Write 0 because you are multiplying by 9 × 10.
32700 ← Write 00 because you are multiplying by 1 × 100.
63765
```

Exercises - Page 10-11a.

1.	188	2.	612	3.	441	4.	205
5.	2576	6.	1584	7.	3784	8.	4935

Exercises - Page 10-11c.

1.	1608	2.	2835	3.	6693
4.	2496	5.	23182	6.	11352

Class Discussion - Page 10-11d.

$13 \times 5 = 65$

$10 \times 10 = 100$

So $\frac{13}{10} \times \frac{5}{10} = \frac{65}{100}$

$\frac{65}{100} = .65$

There is 1 decimal place in 1.3 .

There is 1 decimal place in .5 .

There are 2 decimal places in the product.

$2.56 = \frac{256}{100}$ and $1.5 = \frac{15}{10}$

$256 \times 15 = \underline{3840}$

$100 \times 10 = \underline{1000}$

So $\frac{256}{100} \times \frac{15}{10} = \frac{3840}{1000}$

$= 3.840$

There are 2 decimal places in 2.56 .

There is 1 decimal place in 1.5 .

There are 3 decimal places in 3.840 .

We count all the zeros and put that many zeros in the answer. The number of decimal places in a number like 2.56 just shows how many zeros are in the denominator of the fraction $\frac{256}{100}$. We just count all the decimal places in the numbers we are multiplying and put that many decimal places in the answer.

Exercises - Page 10-11f.

1. (a) .0081 (d) 2.4970 (or 2.497) (g) 1.00 (or 1)
   (b) .00625 (e) .8976 (h) .125
   (c) 144.0 (or 144) (f) .008382 (i) .405

2. (a) 1.56 (c) 50.60 (or 50.6)
   (b) 125.58 (d) 1.864

3. (a) .007 (e) .0275
   (b) 49.8 (f) 3.2596
   (c) .0032 (g) 993.000 (or 993)
   (d) .0680 (or .068)

Lesson 10-12.

Students may be upset to find that the quotient in a problem like $\frac{10}{.05} = ?$ is much larger than the dividend. Even checking the answer by multiplying $(200 \times .05 = 10)$ may not reassure them. Here it is helpful to refer once more to money: How many nickels are there in ten dollars? Show that the smaller the number is that you divide by, the larger the answer is.

Class Discussion - Page 10-12.

If we multiply .5 by <u>10</u> we get 5.

Since there are two decimal places in .25 , we will have to multiply it by <u>100</u> to get the whole number 25 .

$$\frac{75}{.25} \times \frac{100}{100} = \frac{7500}{25.}$$

$$25\overline{)7500}\ \ 300$$

$300 \times .25 = 75.00$ or 75 .

In the problem 6.8 divided by .02, you have to multiply .02 by 100 to get the whole number 2. The name we choose to multiply by is $\frac{100}{100}$.

$$\frac{6.8}{.02} \times \frac{100}{100} = \frac{680}{2}$$

$$= 340$$

Exercises - Page 10-12b.

1. (a) 8
   (b) 6.25
   (c) 5
   (d) 1.[illegible]
   (e) 120
2. (a) 2.5
   (b) .25
   (c) 320
   (d) 30
   (e) .7
   (f) 1.6
3. (a) 6.1
   (b) 9900
   (c) .3
   (d) 4
   (e) 70
   (f) .5
   (g) 4
   (h) 60

Lesson 10-13.

Emphasize the importance of testing whether or not an answer is reasonable by looking at the whole number places.

Class Discussion - Page 10-13.

When you write whole numbers in a column, you line up the digits in ones place. You can always think of a decimal point as being just to the right of the ones place, so you are automatically lining up the decimal points.

Exercises - Page 10-13a.

1. (b) 1.101 (f) .625
   (c) 1.613 (g) .803
   (d) 1.175 (h) 3.634
   (e) 1.0122 (i) 22.625

2. (b) 31.354 (f) 9.883
   (c) .572 (g) 1.738
   (d) .164 (h) .9724
   (e) .778

Lesson 10-14.

This section restates the important ideas developed in the chapter. Assign only the exercises students need.

Class Discussion - Page 10-14.

$\frac{3}{4}$ = .75 and $\frac{1}{2}$ = .5

To change $\frac{1}{4}$ to a decimal, you divide 1 by 4 .

The most important thing for you to remember is that the decimal point separates the whole number part of the decimal from the part that is less than 1. Digits to the left of the decimal point represent whole numbers and the farther they are from the point, the larger the number they represent.

Digits to the right of the decimal point represent numbers that are less than 1 .

In 41.3 + 2.069 , the whole numbers are 41 and 2 .

In 60.8 - 1.79 , the whole numbers are 60 and 1 , so your answer should be about 59 because 60 - 1 is 59 .

In 5.63×2.1, the whole numbers are 5 and 2. Your answer should be about 10 because 5×2 is 10.

In $\frac{25.06}{5.1}$, your answer should be about 5 because 25 divided by 5 is 5.

Exercises - Page 10-14c.

1. (a) .75
   (b) .625
   (c) $.\overline{6}$
   (d) .8

2. (a) $\frac{3}{5}$
   (b) $\frac{3}{25}$
   (c) $\frac{9}{20}$
   (d) $\frac{13}{100}$
   (e) $\frac{2077}{1000}$
   (f) $\frac{31}{10000}$

3. (a) .0197 $\underline{<}$ .2
   (b) 7.6 $\underline{<}$ 7.75
   (c) 40.001 $\underline{<}$ 40.0011
   (d) .987 $\underline{>}$ .9836

4. (a) 80.842
   (b) 54.150

5. (a) 5.4
   (b) 1.478
   (c) 36.879

6. (a) .0391
   (b) 2320
   (c) .42

7. (a) 900
   (b) 3
   (c) .2

Pre-Test Exercises - Page 10-P-1.

1. (a) 49.8
   (b) .002
   (c) 4.682

2. (a) 580
   (b) .462
   (c) 1.2
   (d) 4760
   (e) .375

3. (a) 35.974
   (b) 6.066

4. (a) 1.30
   (b) 4.975

5. (a) 7
   (b) 1300
   (c) .7

6. (a) .5
   (b) .25
   (c) .73
   (d) 1.5
   (e) .375

7. (a) $\frac{7}{20}$ (c) $\frac{2}{5}$
   (b) $\frac{5}{8}$ (d) $\frac{1}{20}$

8. (a) 15 (or 15.00)

   (b) 6 (or 6.00)

   (c) 26.75

9. (a) .039 $\underline{>}$ .0164

   (b) .104 $\underline{<}$ .13

   (c) .0051 $\underline{<}$ .00512

<u>Test</u> - <u>Page</u> <u>10-T-1</u>.

1. (a) .368

   (b) .0029

   (c) .008

2. (a) 2.9

   (b) 5.22

   (c) 82

   (d) .0345

   (e) 57.2

3. (a) 85.647

   (b) 57.21

4. (a) 68.94

   (b) 2.01

5. (a) 25500

   (b) .61

   (c) 16

6. (a) .25
   (b) .75
   (c) .125
   (d) .5
   (e) .89
   (f) .2
   (g) .1
   (h) 1.25

7. (a) $\frac{9}{20}$
   (b) $\frac{3}{8}$
   (c) $\frac{4}{5}$
   (d) $\frac{1}{5}$
   (e) $\frac{1}{25}$
   (f) $\frac{3}{20}$

8. (a) 70 (or 70.00)
   (b) 7.2 (or 7.20)
   (c) 1800
   (d) 23.25
   (e) 39.7

9. (a) .29 $\underline{>}$ .289
   (b) .01604 $\underline{<}$ .016041
   (c) .078 $\underline{>}$ .064

Teacher's Commentary

Chapter 11

PARALLELISM

This chapter is a continuation of the geometry first introduced in Chapter 9, Congruence. Since it is probable that the Congruence chapter was taught in the spring whereas this chapter will be taught in the fall, we have given the students an extensive review of the basic constructions introduced in Congruence.

Through the use of these basic constructions the student's knowledge of geometry is extended to include:

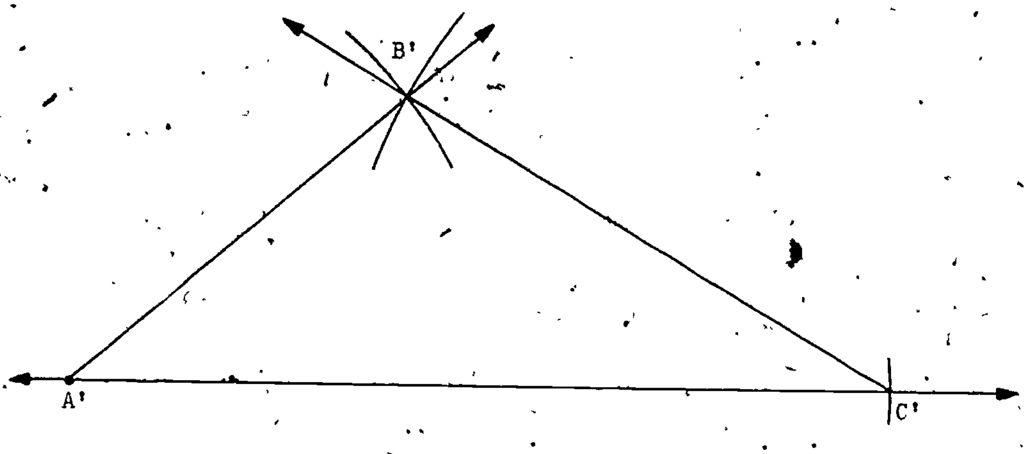
(1) Constructing parallel and perpendicular lines;
(2) Properties of the rectangle and parallelogram;
(3) The use of the congruence properties of triangles to develop some simple geometric proofs.

Other concepts introduced are:

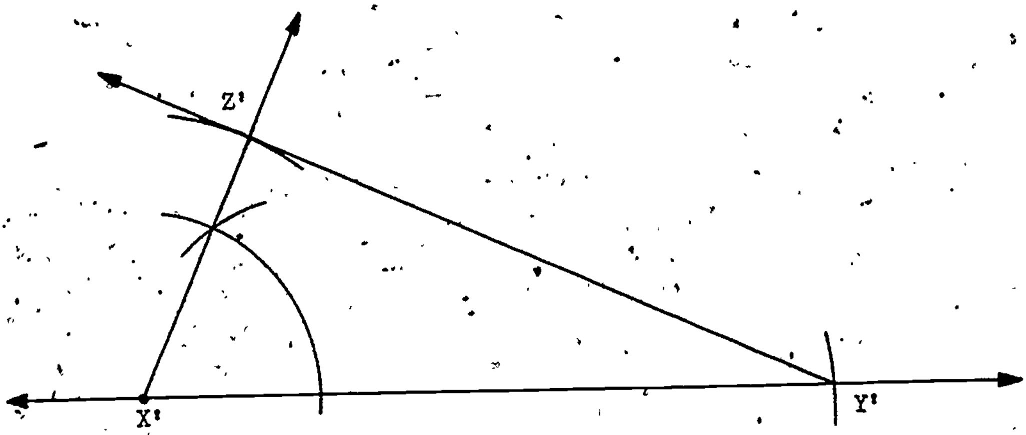
(1) Angle measure;
(2) Vertical angles;
(3) Alternate interior angles;
(4) The sum of the measures of the angles of a triangle is equal to the measure of a straight angle.

Lesson 11-1.

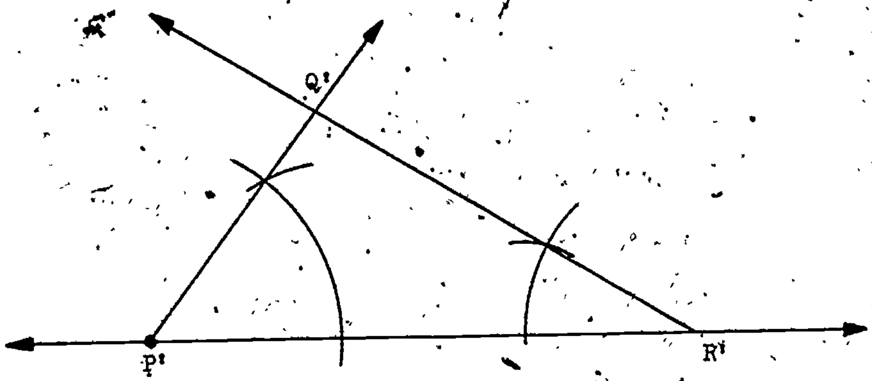
Worksheet - Page 11-1d.



Worksheet - Page 11-1g.

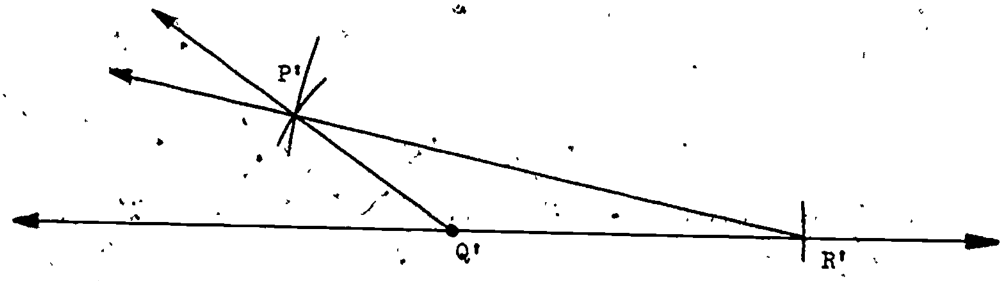


Worksheet - Page 11-1j.

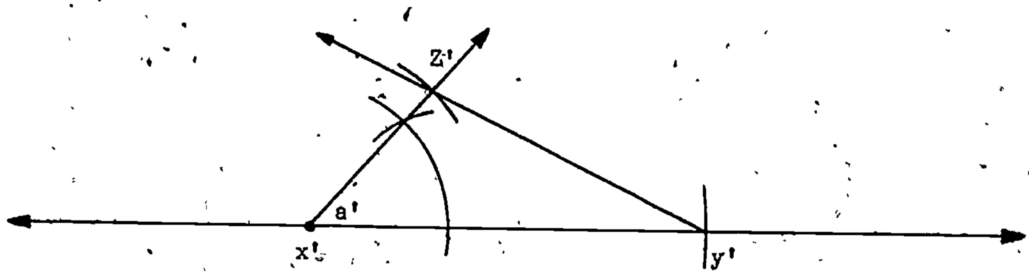


Exercises. - Page 11-1k.

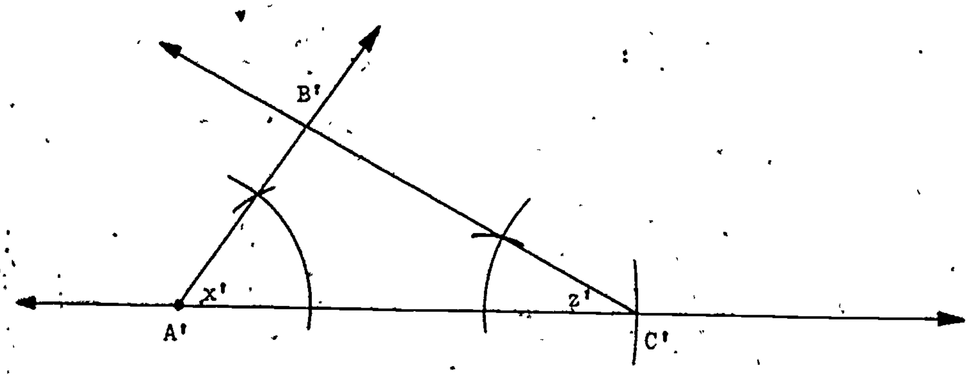
1.



2.

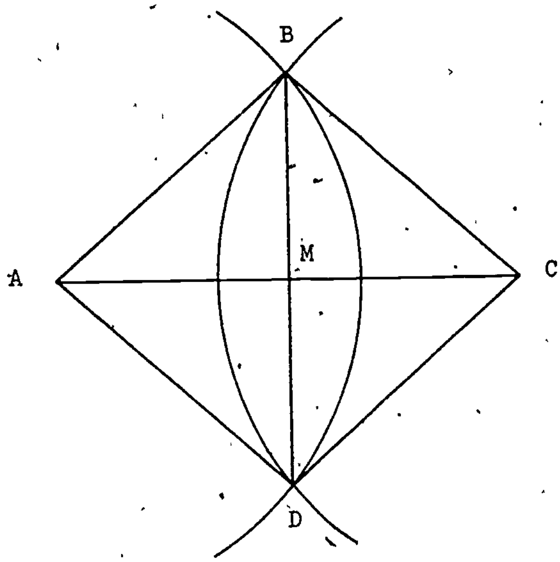


3.

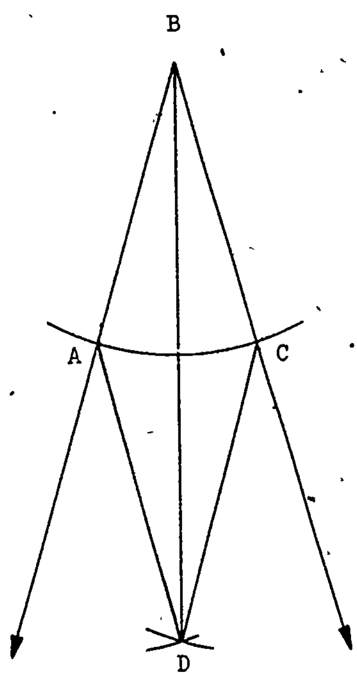


Lesson 11-2.

Worksheet 11-2c.

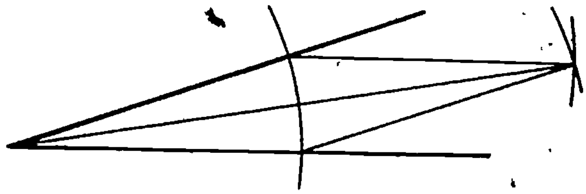


Worksheet 11-2f.

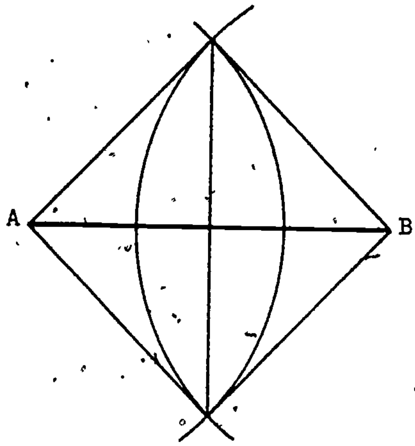


Exercises - Page 11-2g.

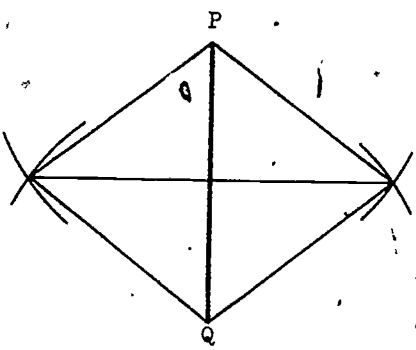
1.



2.

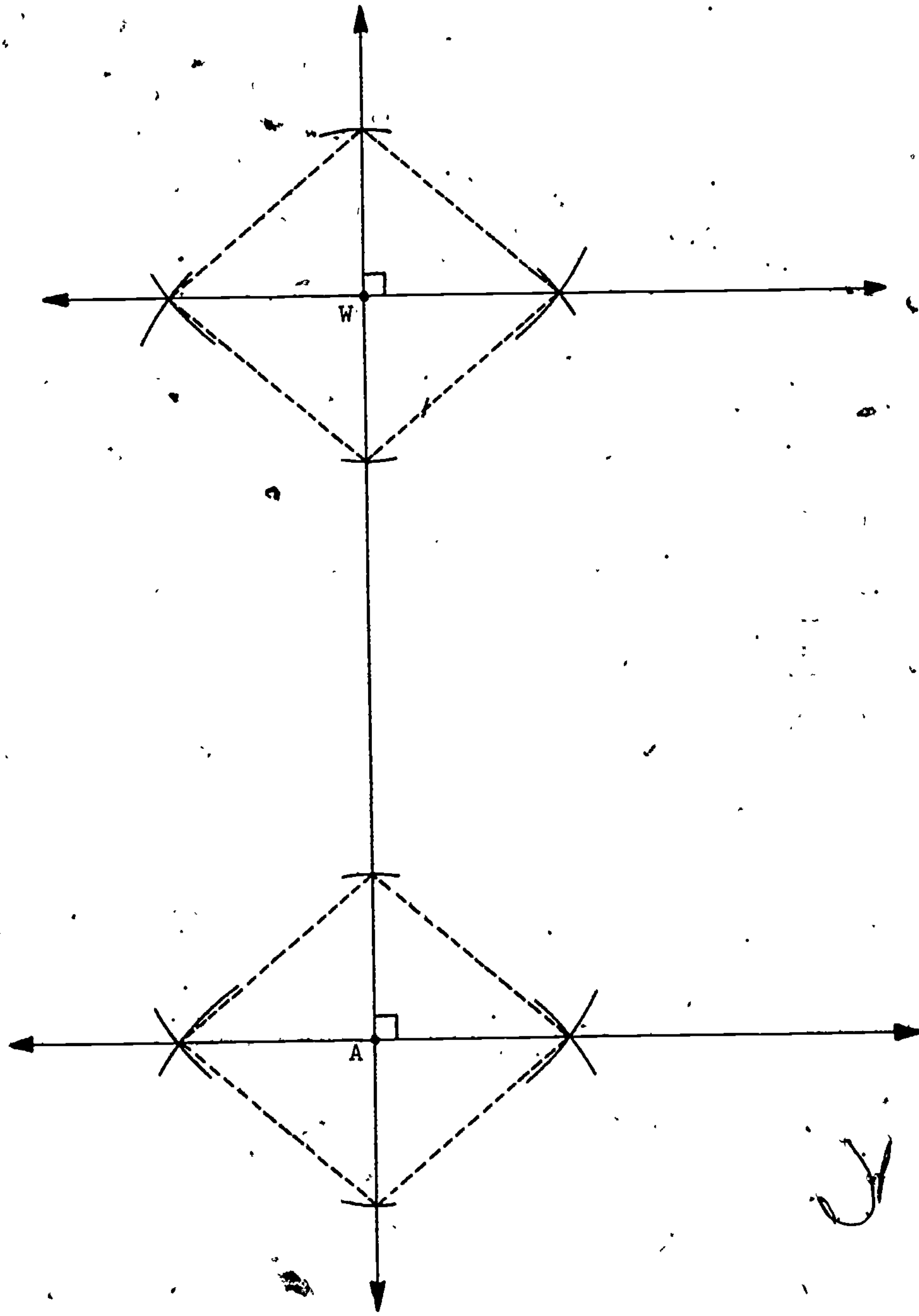


3.



Lesson 11-3.

Worksheet - Page 11-3d.



Exercises - Page 11-3e.

1. (a) Yes

   (b) Yes

   (c) Yes

2. (a) Parallel

   (b) Parallel

3. (a) There is only <u>one</u> line through P parallel to ℓ_1.

   (b) There is only <u>one</u> line through P perpendicular to ℓ_1.

4. BRAINBOOSTER.

   (a) Three

   (b) Four

<u>Lesson 11-4</u>.

Class Discussion - Page 11-4b.

1. $\overline{AB} \cong \underline{\overline{CD}}$

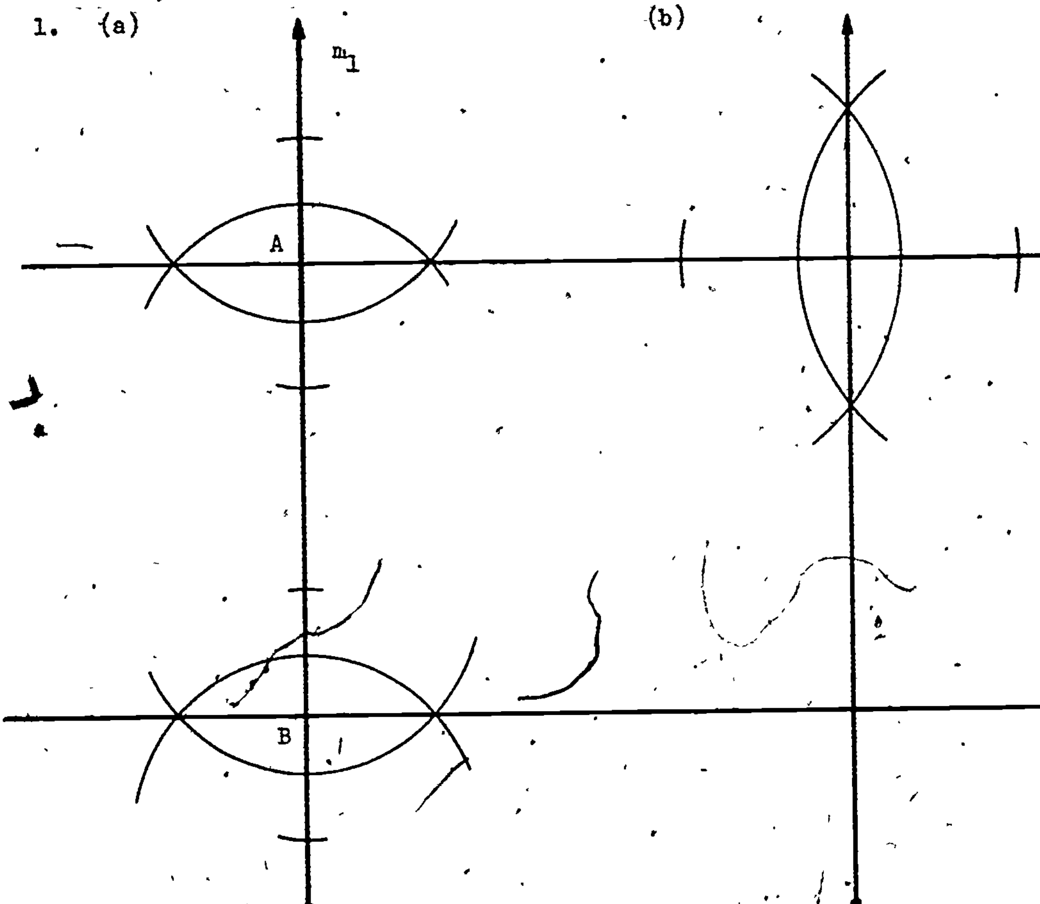
2. $\overline{AD} \cong \underline{\overline{CB}}$

3. $\overline{AC} \cong \underline{\overline{AC}}$

4. Then, by the <u>SSS</u> property of triangles, $\triangle ADC \cong \triangle CBA$.

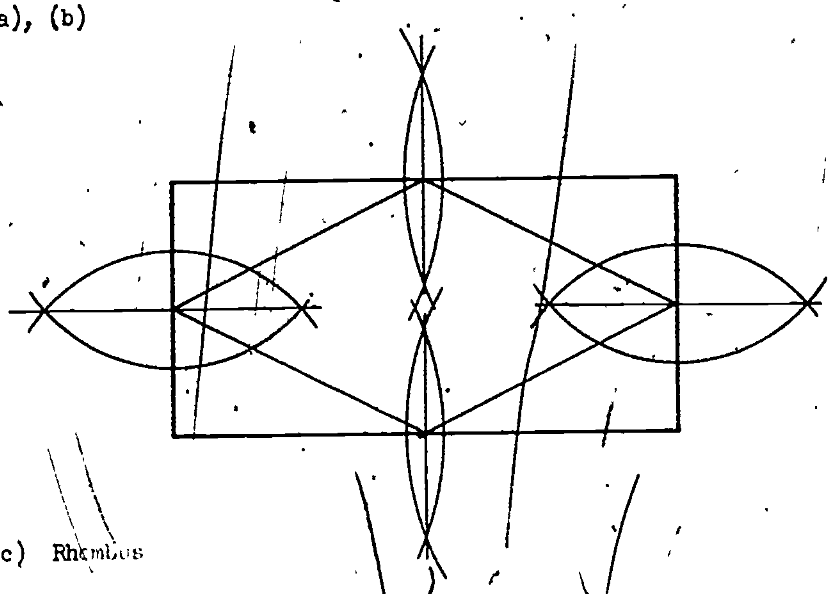
Exercises - Page 11-4c.

1. (a) (b)



(c) Rectangle

2. (a), (b)



(c) Rhombus

3. BRAINBOOSTER.

(a) Since point M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{BM}$.

(b) $m \angle A$ is 90. $m \angle B$ is 90. So $\angle A \cong \angle B$.

(c) $\overline{AD}$ and $\overline{BC}$ are opposite sides of a rectangle. So, $\overline{AD} \cong \overline{BC}$. Therefore $\triangle ADM \cong \triangle BCM$ by SAS congruence property of triangles.

Lesson 11-5.

In this section we are not trying to teach the student how to use a protractor. Our objective is only to explain to the student where we get the number that we assign as the measure of an angle. The student will get a complete treatment on the use of the protractor in Chapter 15, Measurement.

Class Discussion - Page 11-5a.

1. $m \angle x = 40$
2. $m \angle y = 140$
3. $m \angle x + m \angle y = 180$
4. The measure of a straight angle is 180 .
5. (a) $m \angle w = 90$, $m \angle x = 90$
   (b) $m \angle z = 90$
   (c) $m \angle y = 90$
   (d) right
   (e) perpendicular

Exercises - Page 11-5c:

1. (a) $m \angle x = 150$ (b) $m \angle y = 135$
   (c) $m \angle z = 90$ (d) $m \angle z = 60$
   (e) $m \angle z = 50$ (f) $m \angle w = 130$
2. $m \angle w + m \angle x + m \angle y + m \angle z = 180$
3. 90
4. BRAINBOOSTER.

   $m \angle y = 140$

Lesson 11-6.

Class Discussion - Page 11-6.

1. $120 + 60 = 180$
2. $120 + m\angle a = 180$
3. $m\angle a = 180 - (120)$
4. $m\angle a = 60$
5. $60 + m\angle b = 180$
6. $m\angle b = 180 - (60)$
7. $m\angle b = 120$

You may wish to point out to your students that the development of the congruence of vertical angles on page 11-6a is their first introduction to an algebraic proof.

Exercises - Page 11-6b.

1. (a) $m\angle x = 20$
   $m\angle y = 160$

   (b) $m\angle x = 70$
   $m\angle y = 110$

   (c) $m\angle x = 90$
   $m\angle y = 90$

   (d) $m\angle x = 30$
   $m\angle y = 110$
   $m\angle z = 150$

2. 90

3. (a) 90

   (b) $90 + 90 + 90 + 90 = \underline{360}$

4. BRAINBOOSTER.

   $m\angle x + m\angle y + m\angle z = 180$

Lesson 11-7.

Class Discussion - Page 11-7a.

1. $m \angle e = 60$

2. $m \angle c = 60$

3. $m \angle g = 120$

4. $m \angle f = 120$

5. $m \angle d = 120$

6. $\angle b$ and $\angle d$ are vertical angles.

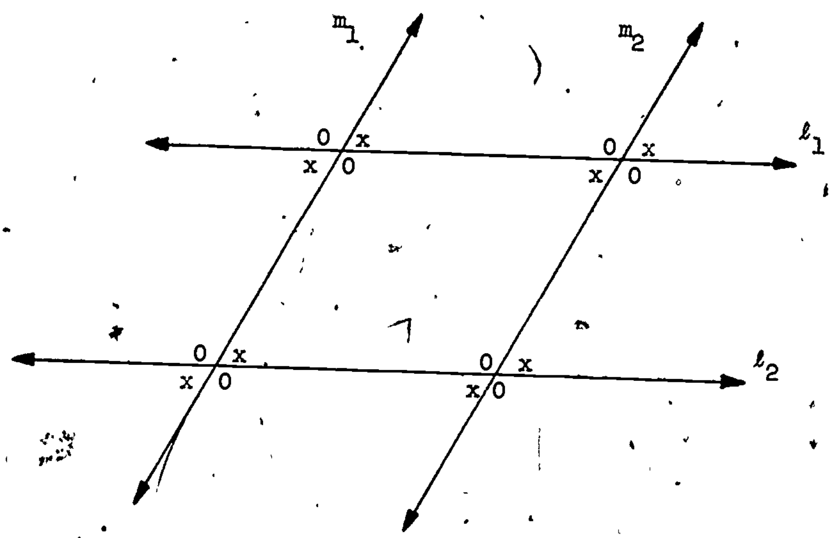
7. $\angle f$ and $\angle d$ are alternate interior angles.

8. (a) $m \angle c = 30$
   (b) $m \angle e = 30$
   (c) $m \angle g = 30$
   (d) $m \angle f = 150$
   (e) $m \angle a = 150$
   (f) $m \angle b = 150$
   (g) $m \angle d = 150$

Exercises - Page 11-7b.

1. (a) $m \angle a = 155$
   (b) $m \angle b = 25$
   (c) $m \angle c = 155$
   (d) $m \angle d = 155$
   (e) $m \angle e = 25$
   (f) $m \angle f = 155$
   (g) $m \angle g = 25$

2.



3. (a) $m \angle y = 30$

 (b) $m \angle x = 60$

 (c) $m \angle z = 60$

4. BRAINBOOSTER.

 (a) $m \angle b = 30$

 (b) $m \angle c = 60$

Lesson 11-8.

The measure of a straight angle is 180.

Exercises - Page 11-8b.

1. (a) $m \angle x = 90$

 (b) $m \angle x = 120$

 (c) $m \angle x = 30$

2. $m \angle x + m \angle y = 90$

3. (a) $m \angle x = 60$

 (b) $m \angle y = 60$

 (c) $m \angle z = 70$

4. BRAINBOOSTER.

 (a) $m \angle x = 105$

 (b) $m \angle y = 30$

Lesson 11-9.

Class Discussion - Page 11-9a.

1. (a) $m \angle a + m \angle b + m \angle c = 180$

 (b) $m \angle a' + m \angle b' + m \angle c' = 180$

 (c) 360

2. (a) yes

 (b) yes

 (c) equal

3. (a) Yes

(b) Yes

(c) Equal

Exercises - Page 11-9b.

1. (a) $m \angle x = 110$

(b) $m \angle w + m \angle y = 70$

(c) 5

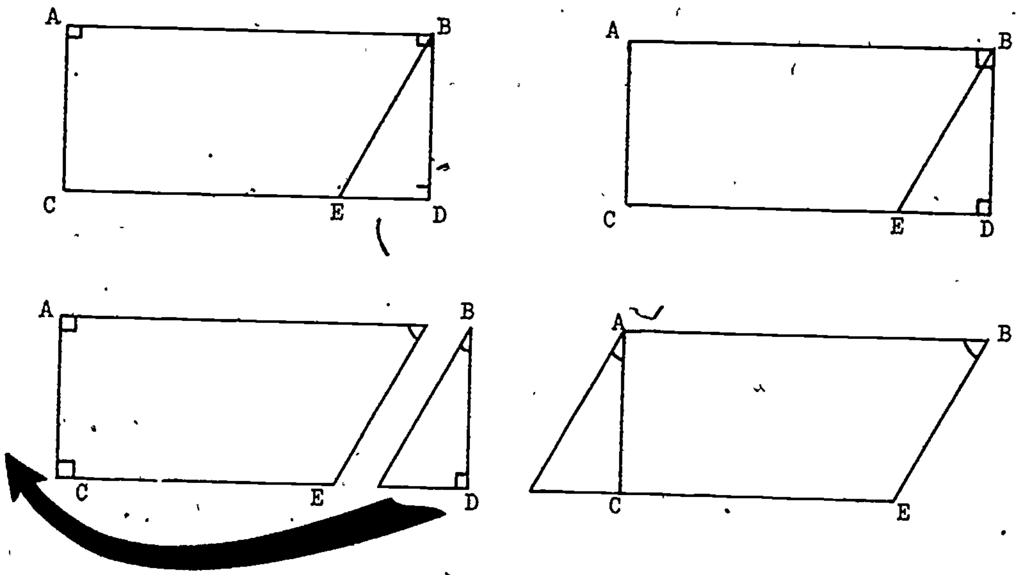
(d) 8

2. (a) $m \angle z = 30$

(b) $m \angle x = 40$

(c) 110

3. (a)



(b) a parallelogram.

4. BRAINBOOSTER.

(a) $\overline{AC} \cong \overline{BD}$ because opposite sides of a parallelogram are congruent.

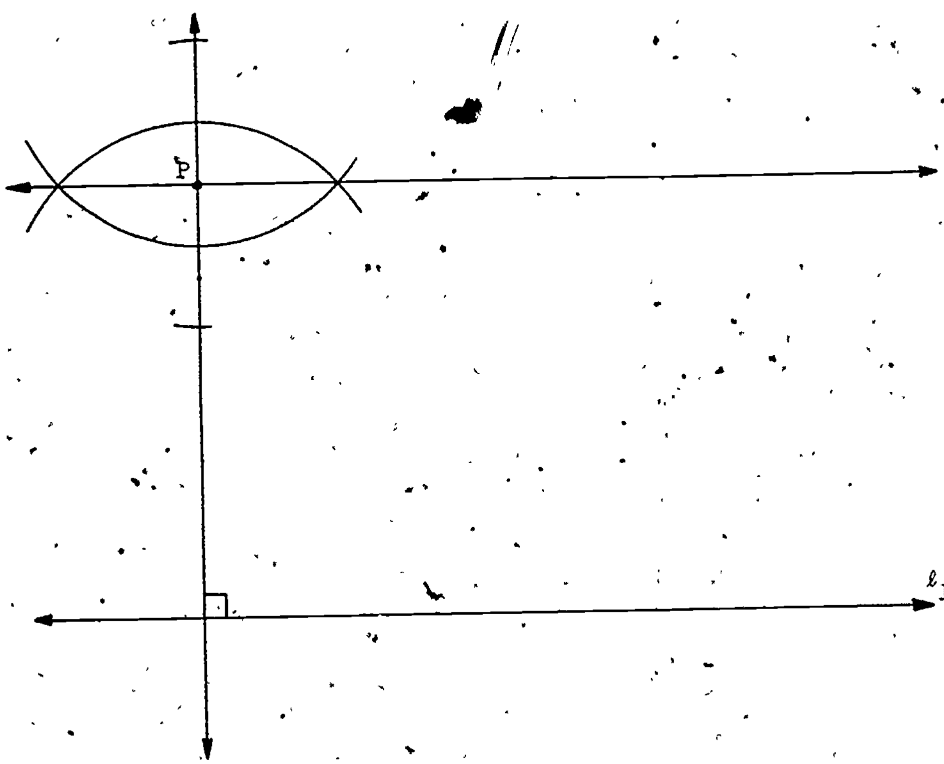
(b) $\overline{CD} \cong \overline{BA}$ because opposite sides of a parallelogram are congruent.

(c) $\overline{AD} \cong \overline{AD}$ because a line segment is congruent to itself.

So, $\triangle ADC \cong \triangle ADB$ by the SSS congruence property of triangles.

Pre-Test Exercises - Page 11-P-1

1.



2. (a) one

(b) one

3. "is parallel to"

4. (a) parallel (also, congruent)

(b) ℓ_1 and ℓ_2

5. (a) $\overline{CD}$

   (b) $\overline{CB}$

   (c) $\overline{DB}$

   (d) $\triangle CDB$

6. SAS

7. 180

8. 90

9. perpendicular

10. (a) $m \angle x = 135$ (c) $m \angle x = 60$

    (b) $m \angle x = 90$ (d) $m \angle x = 155$

11. (a) 180

    (b) 150

12. equal in measure (also, congruent)

13. (a) $m \angle x = 150$
    $m \angle y = 30$

    (b) $m \angle x = 40$
    $m \angle y = 140$

14. 360

15. (a) $\angle a$ and $\angle b$, or $\angle c$ and $\angle d$

    (b) equal

16. (a) $m \angle a = 100$

    (b) $m \angle b = 100$

    (c) $m \angle c = 80$

    (d) $m \angle d = 100$

    (e) $m \angle e = 80$

    (f) $m \angle f = 80$

    (g) $m \angle g = 100$

17. (a) 180

    (b) straight

18. (a) $m \angle x = 90$

(b) $m \angle x = 135$

19. (a) y

(b) z

(c) equal

(d) $\overline{BC}$

(e) $\overline{AB}$

(f) equal

(g) $\overline{BC}$

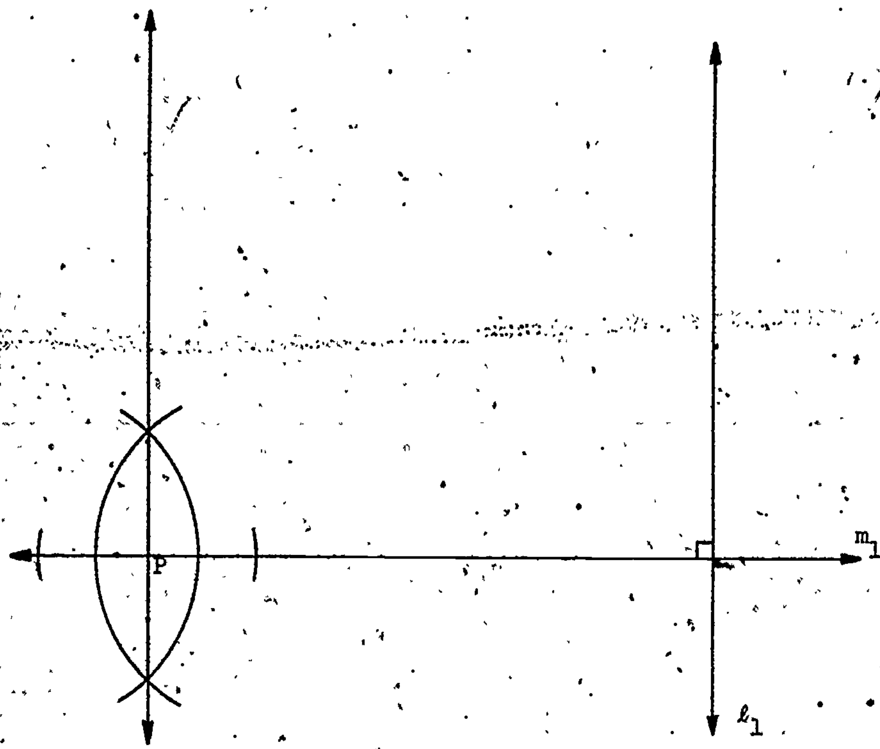
(h) $\overline{DC}$

(i) equal in measure (or, congruent)

(j) 360

Chapter Test - Page 11-T-1.

1.



2. (a) one

(b) one

3. $\ell_1 \parallel \ell_2$

4. (a) parallel (or, congruent)

   (b) same

5. (a) $\overline{BC}$

   (b) $\overline{BA}$

   (c) $\overline{AC}$

   (d) SSS , $\triangle ADC \cong \triangle CBA$

6. (b) $\angle MAD$

   (c) $\overline{CB}$

   (d) SAS , $\triangle MAD \cong \triangle MBC$

7. straight

8. congruent

9. (a) $m \angle x = 150$ (c) $m \angle x = 120$

   (b) $m \angle x = 90$ (d) $m \angle x = 80$

10. $\angle a$ and $\angle c$, or $\angle b$ and $\angle d$

11. (a) $m \angle x = 160$, $m \angle y = 20$ (b) $m \angle x = 100$, $m \angle y = 80$

12. 90

13. $\angle a$ and $\angle b$, or $\angle c$ and $\angle d$

14. (a) $m \angle a = 60$

    (b) $m \angle b = 120$

    (c) $m \angle c = 60$

    (d) $m \angle d = 60$

    (e) $m \angle e = 120$

    (f) $m \angle f = 60$

    (g) $m \angle g = 120$

15. 180

16. (a) $m\angle x = 120$

    (b) $m\angle x = 20$

17. (a) $\angle x$ and $\angle z$, or $\angle y$ and $\angle w$

    (b) $\overline{AB}$ and $\overline{CD}$, or $\overline{BC}$ and $\overline{AD}$

    (c) 360

Teacher's Commentary

Chapter 12

SIMILARITY

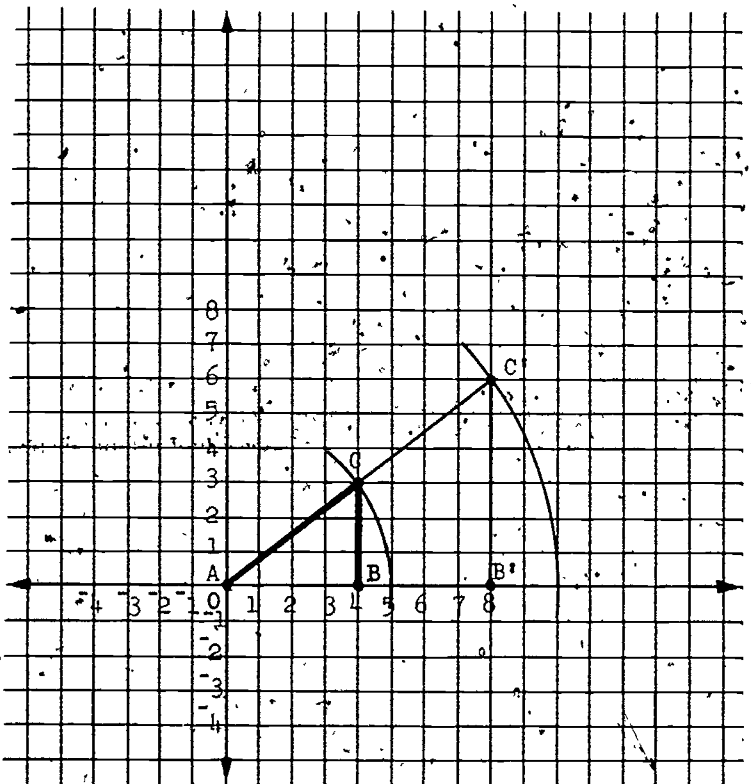
Many students will have a strong intuitive recognition that many objects appear to have the same "shape". In this chapter we examine some of the mathematical bases for this property of "same shapeness". In particular, we will concentrate on similar triangles and the equality of ratios of lengths of corresponding sides. Use is made of similar triangles to reinforce techniques in applying the comparison property and in solving simple linear equations. We also call upon the idea of similarity to introduce percent through a geometric presentation.

Lesson 12-1.

Point out to the students that for any pair of similar triangles there are always two scale factors: one for stretching and one for shrinking. In this lesson, the scale factor for stretching will always be a number greater than one and the scale factor for shrinking will always be a fraction between zero and one. These two scale factors are the reciprocals of each other. If the students have difficulty determining the shrinking scale factor (and they will) then they can always determine the stretching scale factor and take its reciprocal.

1. On the coordinate plane below plot and label the points A(0,0), B(4,0), C(4,3).

2.



3. A right triangle.

4. (a) The length of side $\overline{AB}$ is 4 units.

(b) The length of side $\overline{BC}$ is 3 units.

(c) Is there any way we can count the number of units in the length of side $\overline{AC}$? no .

5. (a) and (b) See graph.

(c) 5

(d) 5

6. A'(0,0)

B'(8,0)

C'(8,6)

7. See graph.

8. See graph.

9. A right triangle.

10. (a) The length of side $\overline{A'B'}$ is $\underline{8}$ units.

(b) The length of side $\overline{B'C'}$ is $\underline{6}$ units.

(c) How many units in length would you guess side $\overline{A'C'}$ to be?
<u>Students should guess about 10 .</u>

(d) The length of $\overline{A'C'}$ is 10 .

11. (a) The length of side $\overline{A'B'}$ is $\underline{2}$ times the length of side $\overline{AB}$.

(b) The length of side $\overline{B'C'}$ is $\underline{2}$ times the length of side $\overline{BC}$.

(c) The length of side $\overline{A'C'}$ is $\underline{2}$ times the length of side $\overline{AC}$.

Exercises - Page 12-1d.

1. (a) $\frac{1}{3}$ and 3

(b) 2 and $\frac{1}{2}$

(c) $\frac{1}{4}$ and 4

2. (a) The scale factor is $\underline{2}$.
The length of $\overline{A'C'}$ is $\underline{2 \cdot 7}$ or $\underline{14}$.

(b) The scale factor is $\underline{\frac{1}{2}}$.
The length of $\overline{BC}$ is $\underline{\frac{1}{2} \cdot 14}$ or $\underline{7}$.

<u>Lesson 12-2.</u>

Class Discussion - Page 12-2.

1. m $\angle$ ACB is 90

2. 180

3. m $\angle$ x is 30.

4. m $\angle$ AC'B' is 90

5. m $\angle$ x' is 30

6. $\triangle ABC$ $\triangle AB'C'$

(a) m $\angle$ A = 60 m $\angle$ A = 60

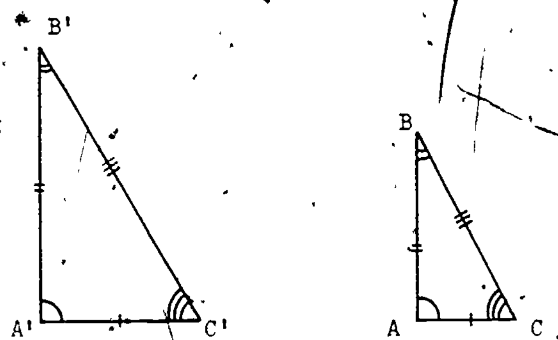
(b) m $\angle$ ACB = 90 m $\angle$ AC'B' = 90

(c) m $\angle$ x = 30 m $\angle$ x' = 30

(d) If two triangles are similar the corresponding angles have equal measure.

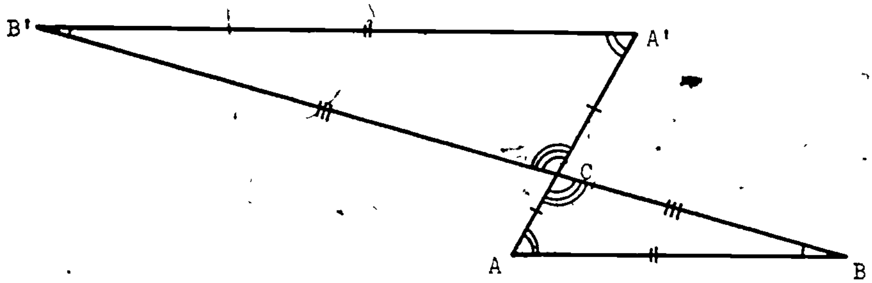
Exercises - Page 12-2b.

1.

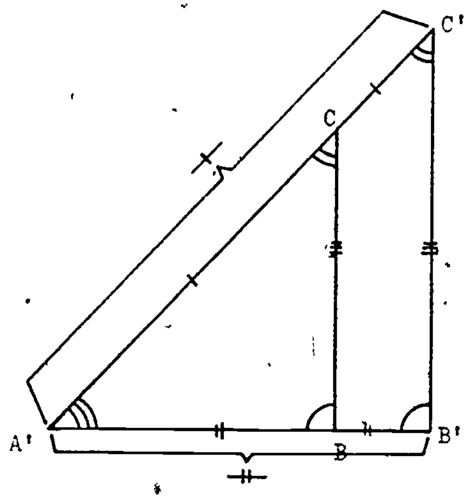


2.

3.



4.



5. (a) (i) The scale factor is 3.

(ii) The length of $\overline{C'B'}$ is $3 \cdot 5$ or 15.

(iii) The length of $\overline{A'B'}$ is $3 \cdot 5$ or 15.

(b) (i) The scale factor is $\frac{1}{3}$.

(ii) The length of $\overline{A'C'}$ is $\frac{1}{3} \cdot 6$ or 2.

(iii) The length of $\overline{B'C'}$ is $\frac{1}{3} \cdot 9$ or 3.

6. BRAINBOOSTER.

(i) The scale factor is 2.

(ii) The length of side $\overline{AC}$ is $2 \cdot 2$ or 4.

(iii) The length of side $\overline{BC}$ is $2 \cdot 2\frac{1}{2}$ or 5.

Lesson 12-3.

In this lesson the student will be called upon to use the Comparison Property and to find the solution to simple linear equations. Since it may have been some time since the student has had the opportunity to use the Comparison Property or to solve equations, a short review may be well worthwhile before you begin this lesson. In particular, remind the student by using numerical examples that

$$\text{if } \frac{a}{b} = \frac{c}{d}, \ b \text{ and } d \neq 0, \text{ then } a \cdot d = b \cdot c \ .$$

Also, to solve an equation of the form $a \cdot x = b$, you multiply both sides of the equation by the reciprocal of a. Demonstrate this procedure by solving a few equations of this form for the student.

Class Discussion - Page 12-3a.

1. $\dfrac{\text{length of } \overline{A'B'}}{\text{length of } \overline{AB}} = \dfrac{6}{3}$

2. $\dfrac{\text{length of } \overline{A'C'}}{\text{length of } \overline{AC}} = \dfrac{x}{5}$

3. $\dfrac{x}{5} = \dfrac{6}{3}$

4. $\underline{3} \cdot x = \underline{5} \cdot 6$

5. $\underline{(\tfrac{1}{3})} \cdot 3 \cdot x = \underline{(\tfrac{1}{3})} \cdot 30$

   $x = \underline{10}$

6. length of $\overline{A'C'}$ is 10

7. $\dfrac{\text{length of } \overline{A'B'}}{\text{length of } \overline{AB}} = \dfrac{6}{3}$

8. $\dfrac{\text{length of } \overline{B'C'}}{\text{length of. } \overline{BC}} - \dfrac{y}{6}$

9. $\dfrac{y}{6} = \dfrac{6}{3}$

10. $\underline{3} \cdot y = \underline{6} \cdot \underline{6}$

11. $\frac{1}{3}$

$$\left(\frac{1}{3}\right) \cdot 3 \cdot y = \left(\frac{1}{3}\right) \cdot 36$$

$$y = 12$$

12. length of $\overline{B'C'}$ is <u>12</u>.

Exercises - Page 12-3c.

1. $x = 21$

2. $x = 2$

3. $x = 20$

   $y = 15$

4. $x = 4$

   $y = 5$

5. $x = 4$

6. Scale factor for stretching is $2\frac{1}{2}$ or $\frac{5}{2}$.

7. BRAINBOOSTER.

$$\frac{\text{length of the pole's shadow}}{\text{length of the friend's shadow}} = \frac{24}{4}$$

$$\frac{\text{height of the pole}}{\text{height of the friend}} = \frac{x}{6}$$

$$\frac{x}{6} = \frac{24}{4}$$

$$4 \cdot x = 6 \cdot 24$$

$x = 36$ The pole is 36 feet high.

<u>Lesson 12-4</u>.

It is not the intent of this lesson to teach students how a photo enlarger works. Our intent is to show a practical application of similarity in a setting most students find interesting. The development is not crucial to this chapter.

Exercises - Page 12-4d.

1. Distance from top = <u>6</u> inches.
2. Distance from top = <u>4</u> inches.
3. Distance from top = <u>2</u> inches.
4. BRAINBOOSTER.

   Distance from top = $\underline{4\frac{4}{5}}$ <u>(or 4.8)</u> inches.

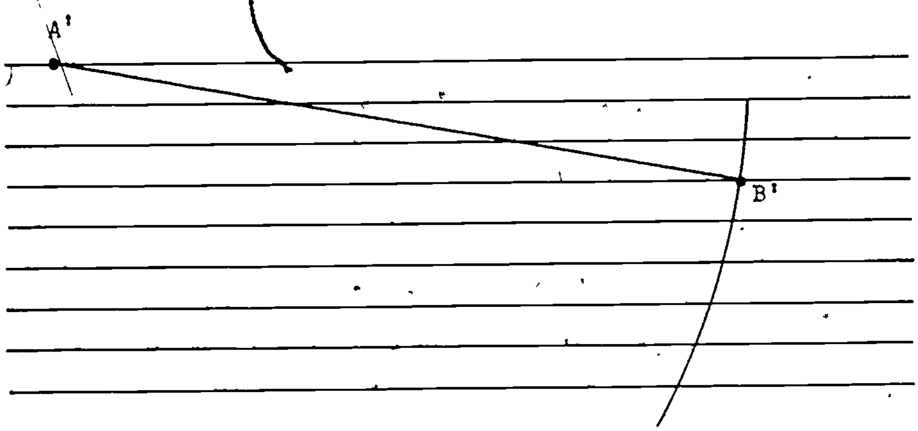
<u>Lesson 12-5</u>.

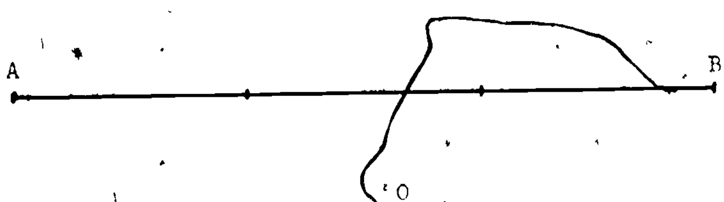
Class Discussion - Page 12-5a.

This exercise is designed to convince the student that if a line is drawn through equally spaced parallel lines then the segments cut off by the parallel lines are all congruent. The answers to all the problems should be <u>yes</u>.

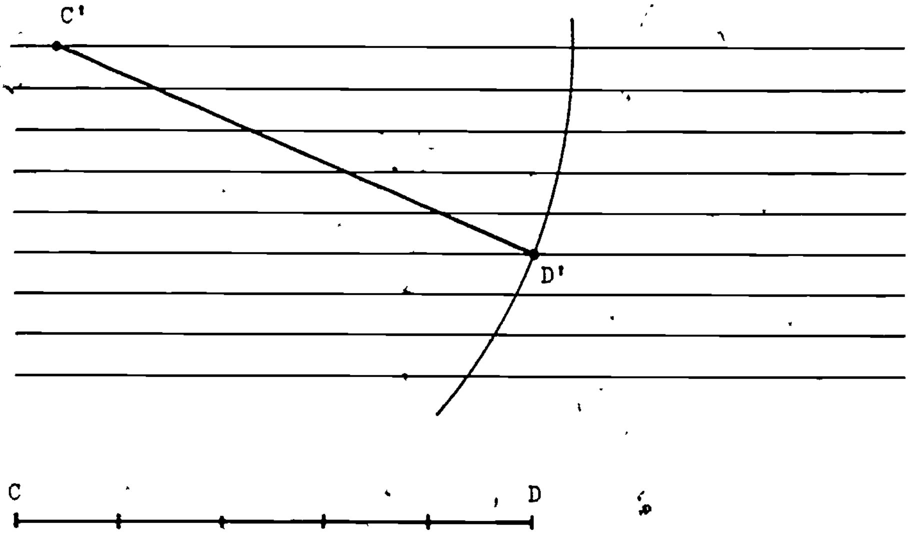
Exercises - Page 12-5e.

1.

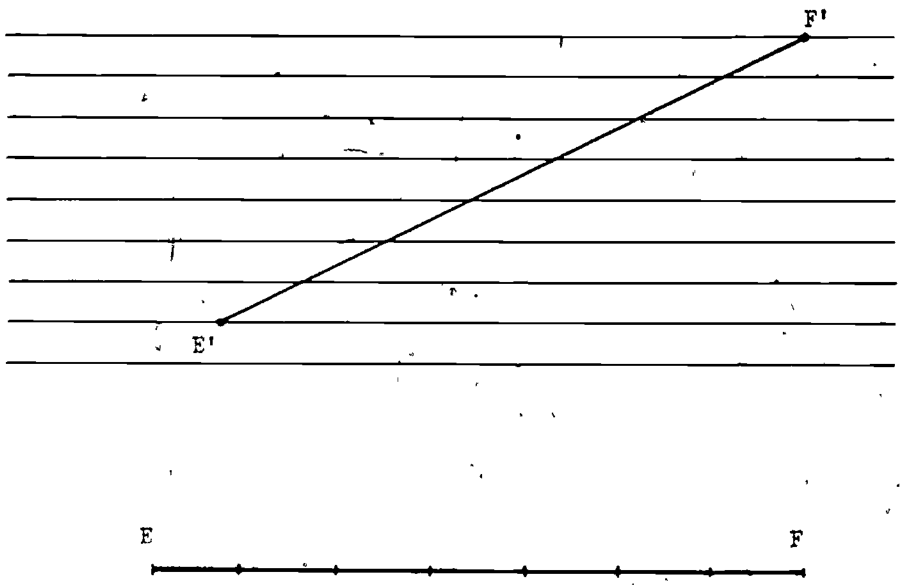




2.



3.



Lesson 12-6.

Class Discussion - Page 12-6b.

1. $\frac{20}{100}$

2. $\frac{x}{80}$

3. $\frac{x}{80} = \frac{20}{100}$

4. $\underline{100} \cdot x = \underline{80} \cdot \underline{20}$

5. $\frac{1}{100}$

6. $\underline{(\frac{1}{100})} \cdot 100 \cdot x = \underline{(\frac{1}{100})} \cdot 1600$

$x = 16$

Exercises - Page 12-6b.

1. (a) $\frac{x}{60} = \frac{50}{100}$

(b) $100 \cdot x = 60 \cdot 50$

$100 \cdot x = 3000$

(c) $\frac{1}{100}$

(d) $(\frac{1}{100}) \cdot 100 \cdot x = (\frac{1}{100}) \cdot 3000$

$x = 30$

2. (a) $\frac{x}{30} = \frac{100}{50}$

(b) $50 \cdot x = 30 \cdot 100$

$50 \cdot x = 3000$

(c) $\frac{1}{50}$

(d) $(\frac{1}{50}) \cdot 50 \cdot x = (\frac{1}{50}) \cdot 3000$

$x = 60$

3. $$\frac{x}{100} = \frac{30}{60}$$

$$60 \cdot x = 100 \cdot 30$$

$$(\frac{1}{60}) \cdot 60 \cdot x = (\frac{1}{60}) \cdot 3000$$

$$x = 50$$

4. $$\frac{x}{100} = \frac{20}{80}$$

$$80 \cdot x = 100 \cdot 20$$

$$(\frac{1}{80}) \cdot 80 \cdot x = (\frac{1}{80}) \cdot 2000$$

$$x = 25$$

5. $$\frac{x}{60} = \frac{80}{100}$$

$$100 \cdot x = 60 \cdot 80$$

$$(\frac{1}{100}) \cdot 100 \cdot x = (\frac{1}{100}) \cdot 4800$$

$$x = 48$$

6. $$\frac{x}{12} = \frac{100}{30}$$

$$30 \cdot x = 12 \cdot 100$$

$$(\frac{1}{30}) \cdot 30 \cdot x = (\frac{1}{30}) \cdot 1200$$

$$x = 40$$

Lesson 12-7.

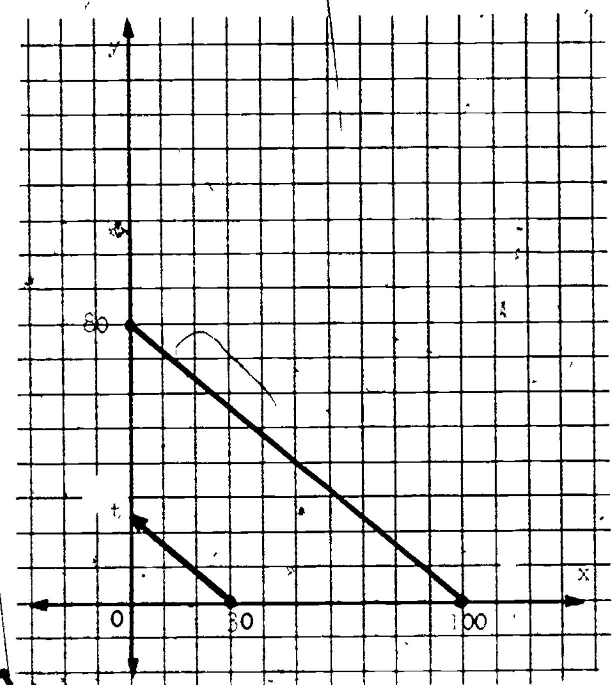
We have chosen to introduce percent through the use of similar triangles for several reasons. First, the visual display helps the student set up the correct proportion. Second, with a little practice the student learns to make a fairly close approximation to the solution of a percent problem against which he can "check" his arithmetic solution. Finally, in practice this approach has been successful when all other approaches have failed.

While it is desirable that all students eventually use an arithmetic approach to solving percent problems, many students at this stage of their

development are not ready to do so. It is recommended for those students who are obviously not ready for this transition that they be encouraged to use the graphical method presented here.

Class Discussion - Page 12-7b.

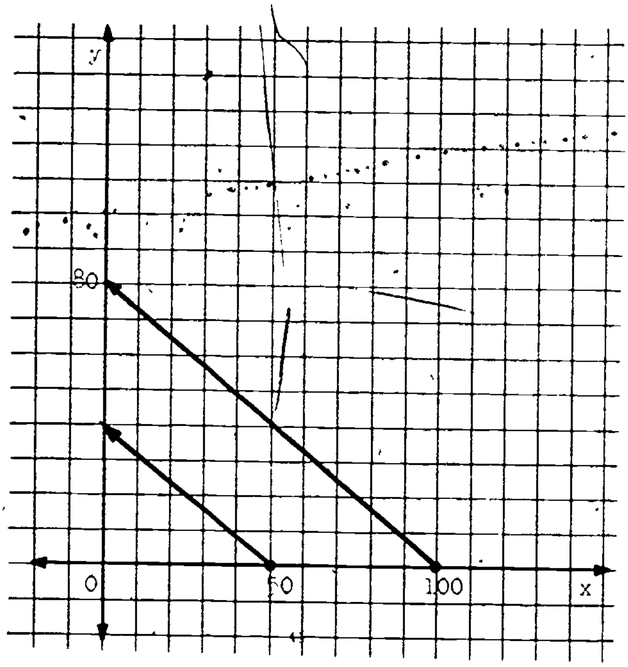
(a) through (d)



(e) A reasonable guess would be about 25.

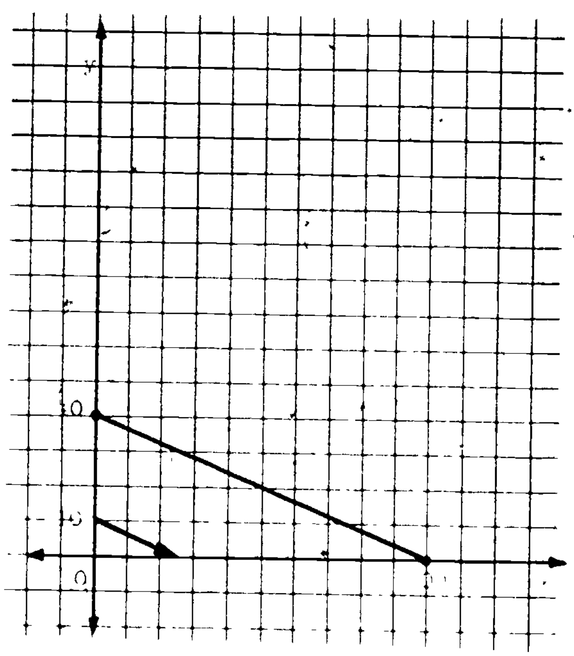
Exercises - Page 12-7d.

1.



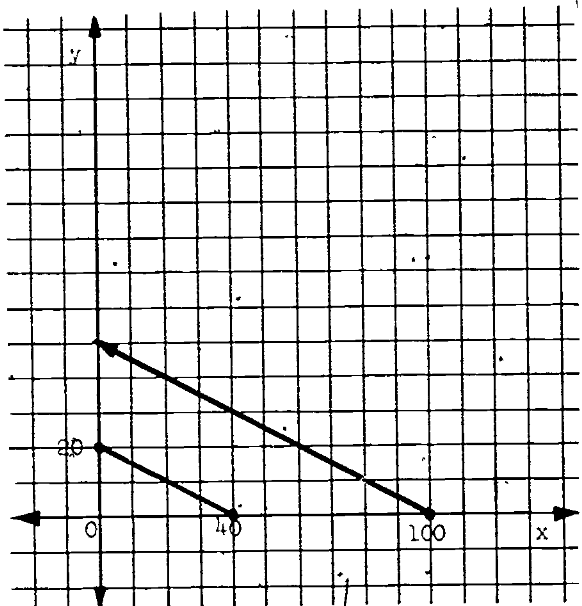
50% of 80 is about 40 .

2.



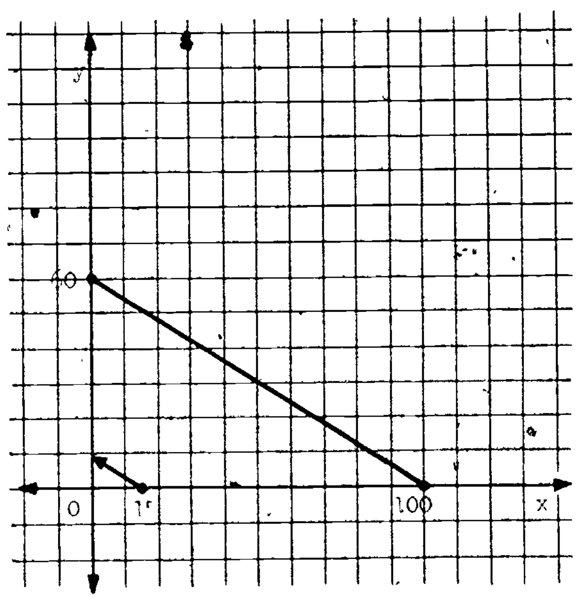
10 is about 25% of 40 .

3.



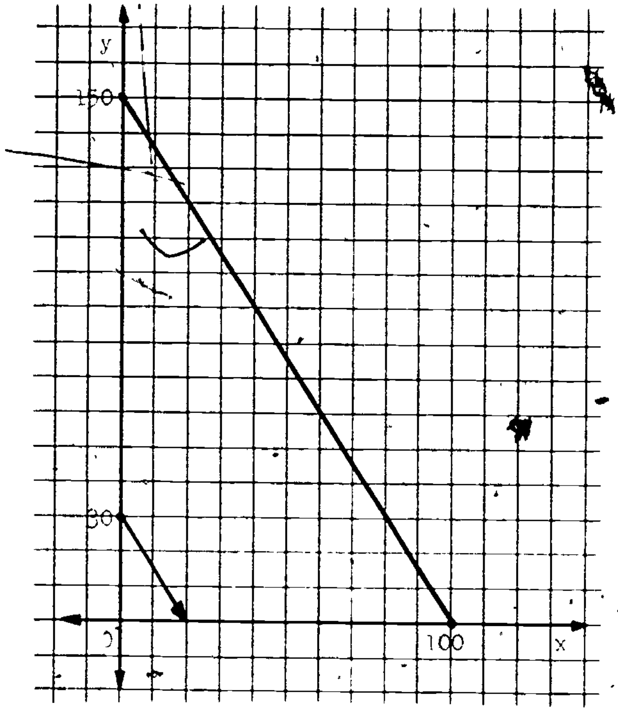
40% of about 50 is 20.

4.



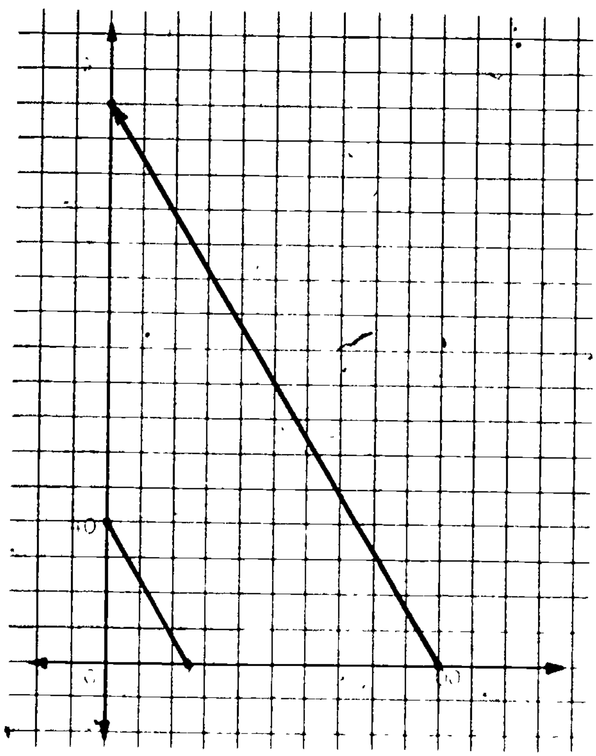
15% of 60 is about 9.

5.



30 is about 20% of 150.

6.

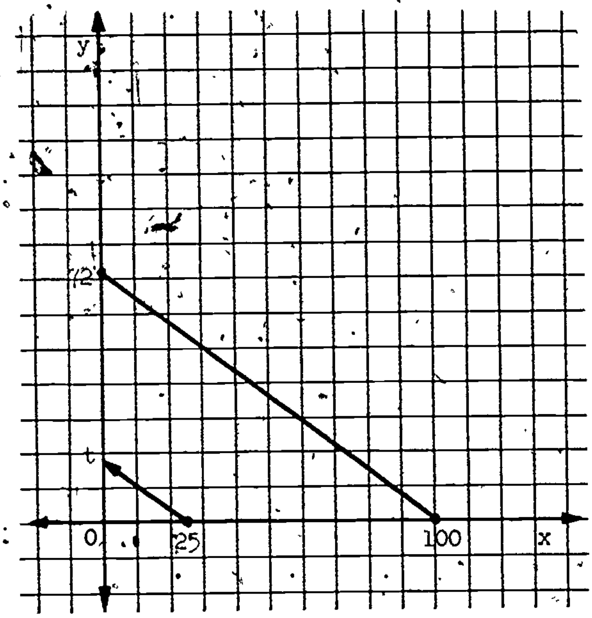


[illegible]% of about [illegible]0 is [illegible]0.

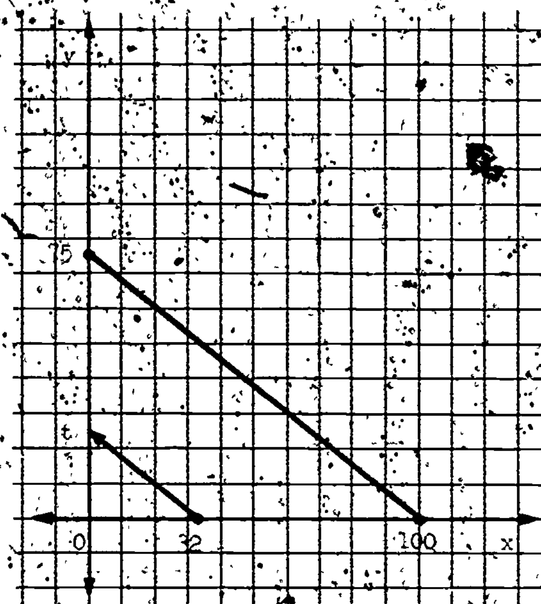
Lesson 12-8

Exercises - Page 12-8a.

1.



2.



32% of $75 =$ 24

(Arithmetic solution)

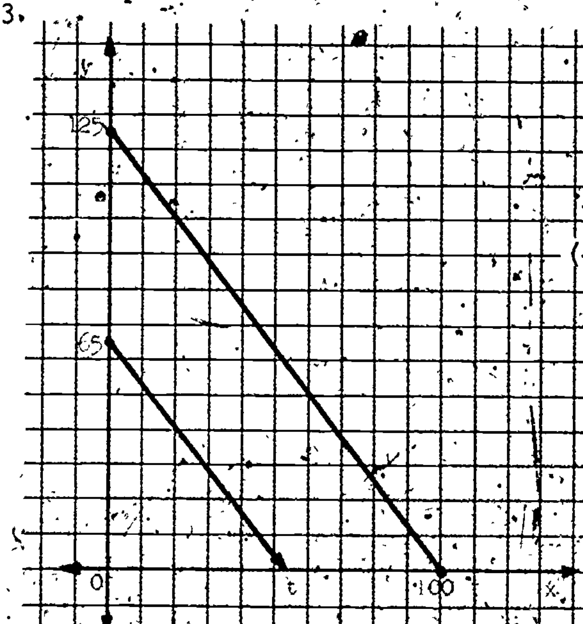
$$\frac{t}{75} = \frac{32}{100}$$

$$100 \cdot t = 75 \cdot 32$$

$$\left(\frac{1}{100}\right) \cdot 100 \cdot t = \left(\frac{1}{100}\right) \cdot 2400$$

$$t = 24$$

3.



52% of 125 is 65.

(Arithmetic solution)

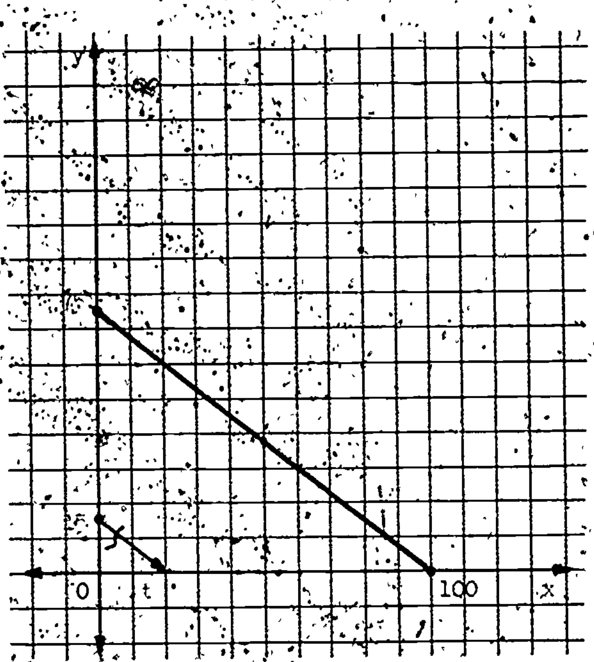
$$\frac{t}{100} = \frac{65}{125}$$

$$125 \cdot t = 100 \cdot 65$$

$$\left(\frac{1}{125}\right) \cdot 125 \cdot t = \left(\frac{1}{125}\right) \cdot 6500$$

$$t = 52$$

4.



(Arithmetic solution)

$$\frac{t}{100} = \frac{15}{75}$$

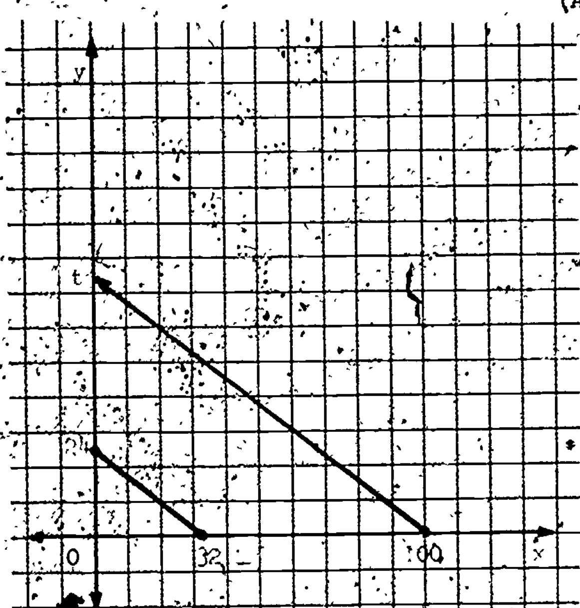
$$75 \cdot t = 100 \cdot 15$$

$$\left(\frac{1}{75}\right) \cdot 75 \cdot t = \left(\frac{1}{75}\right) \cdot 1500$$

$$t = 20$$

20% of 75 is 15

5.



(Arithmetic solution)

$$\frac{32}{100} = \frac{24}{t}$$

$$32 \cdot t = 24 \cdot 100$$

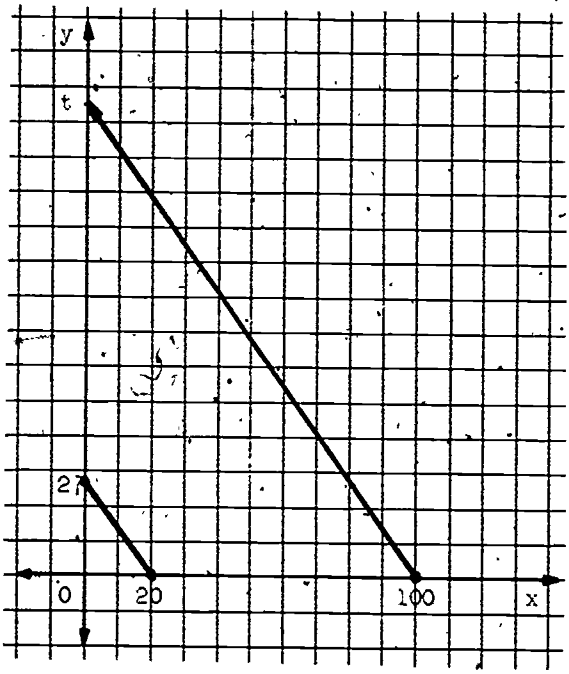
$$32 \cdot t = 2400$$

$$\left(\frac{1}{32}\right) \cdot 32 \cdot t = \left(\frac{1}{32}\right) \cdot 2400$$

$$t = 75$$

32% of 75 is 24

6.



(Arithmetic solution)

$$\frac{t}{27} = \frac{100}{20}$$

$$20 \cdot t = 27 \cdot 100$$

$$\left(\frac{1}{20}\right)20 \cdot t = \left(\frac{1}{20}\right) \cdot 2700$$

$$t = 135$$

20% of 135 is 27.

Pre-Test Exercises - Page 12-P-1.

1. (Section 12-1)

(a) 3, $\frac{1}{3}$

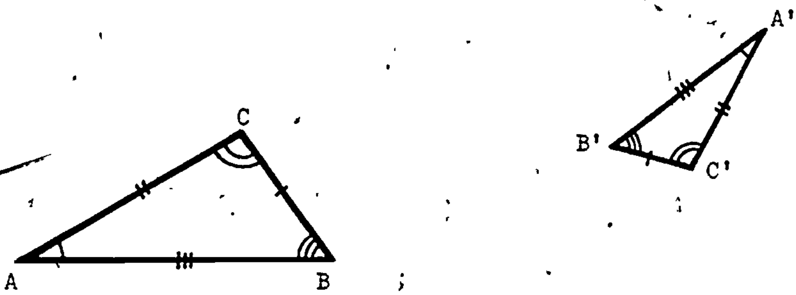
(b) 2, $\frac{1}{2}$

2. (Section 12-1)

(a) 2

(b) $m\,\overline{A'C'} = 10$

3. (Section 12-2)



4. (a) x = 12

(b) x = 4

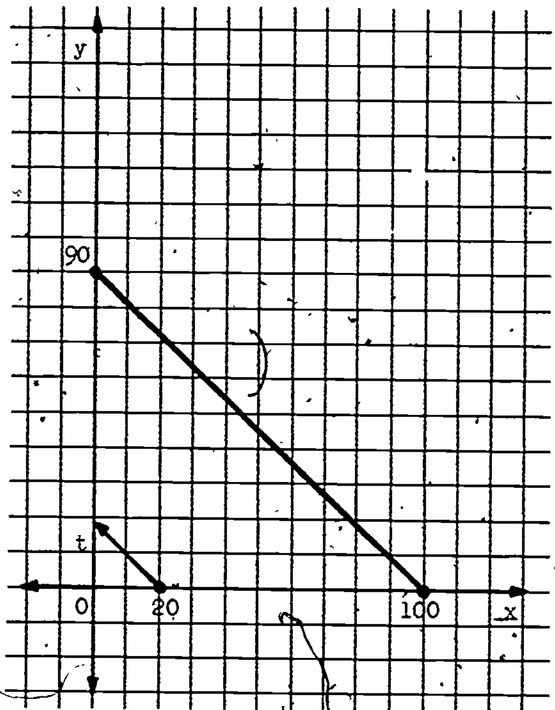
5. $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ are equal in measure.

6. x = 15

7. 25%

8. t is about 25

9.



20% of 90 is 18.

Any estimate between 15 and 23 should be an acceptable answer.

10. (a) x = 30

(b) x = 90

(c) x = 50

11. (a) 3 is 25% of 12.

(b) 15% of 80 is 12.

(c) 30% of 120 is 36.

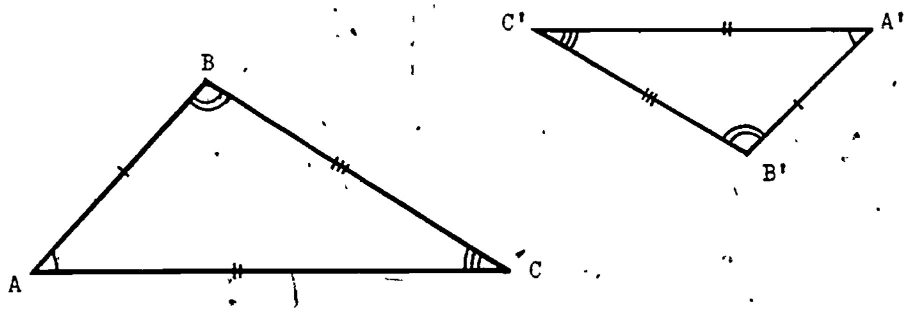
Test - Page 12-T-1.

1. (a) 2, $\frac{1}{2}$

   (b) 3, $\frac{1}{3}$

2. (a) Scale factor is $\underline{\frac{1}{2}}$. (b) m $\overline{A'C'}$ = $\underline{13}$.

3.



4. (a) x = 4

   (b) x = 3

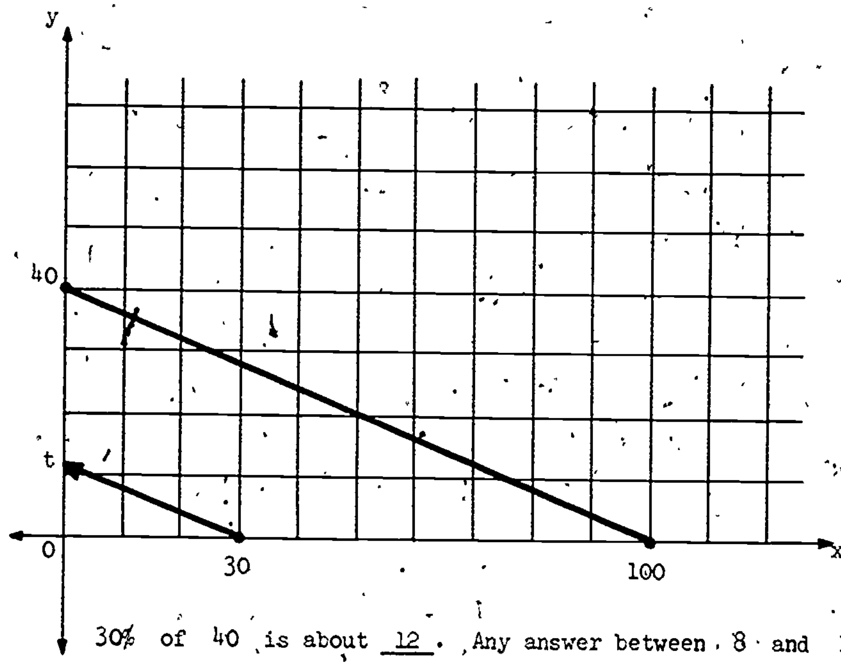
5. $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ are equal in measure.

6. x = 16

7. 50%

8. t is about $\underline{50}$.

9.



30% of 40 is about 12. Any answer between 8 and 15 should be acceptable.

10. (a) x = 40

(b) x = 180

(c) x = 25

11. (a) 5 is 25% of 20.

(b) 25% of 80 is 20.

(c) 40% of 90 is 36.

Chapter 13

MORE ABOUT RATIONAL NUMBERS

When this chapter has been completed it is hoped that students will be able to add, subtract, multiply, and divide rational numbers. (Mixed numerals are more appropriately discussed as a topic of measurement and are not covered here.)

The objectives are:

(1) to review briefly the concepts of Chapter 5, The Integers, including comparison, addition, subtraction, and multiplication;

(2) to teach division of integers, including negative integers and to reemphasize the connection between multiplication and division;

(3) to teach students: (a) that $\frac{^-1}{2}$, $\frac{1}{^-2}$, and $^-(\frac{1}{2})$ all name the same number, with $\frac{^-1}{2}$ the most useful form and (b) that $\frac{^-1}{^-4}$ and $\frac{1}{4}$ name the same number, with $\frac{1}{4}$ the most useful form;

(4) to review addition and subtraction involving fractions with like denominators;

(5) to teach the meaning of <u>multiple</u>, <u>common multiple</u>, and <u>least common multiple</u> and to provide a technique for finding the L.C.M.;

(6) to teach two ways of adding rational numbers: (a) using a flow chart and (b) finding the least common denominator;

(7) to reinforce the idea that when you subtract you add the opposite of the subtrahend;

(8) to introduce negative exponents with powers of 10 and show how to multiply or divide powers of 10 by adding or subtracting exponents;

(9) to reinforce the idea of the relationship between fractions and decimals in powers of 10;

(10) to provide a tool with which students can write any number using scientific notation or reverse the process;

(11) to show the relationship of fractions, decimals, and percents.

<u>Lesson</u> <u>13-1</u>.

Class Discussion - Page 13-1.

1. Numbers to the right of zero are <u>positive</u> integers
   To the left of zero are the <u>negative</u> integers.

$^-10 \quad ^-9 \quad ^-8 \quad ^-7 \quad ^-6 \quad ^-5 \quad ^-4 \quad ^-3 \quad ^-2 \quad ^-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

2. 5 units to the right of zero: <u>5</u>
   6 units to the left of zero: <u>$^-6$</u>

3. 0

4. 3 units
   To the <u>right</u>

5. 4 units
   To the <u>right</u>
   4

6. 3 units
   To the <u>left</u>
   $^-3$

7. 3

8. (a) $1 + 4 =$ <u>5</u> (d) $4 + {}^-5 =$ <u>$^-1$</u>
   (c) $^-1 + 5 =$ <u>4</u> (b) $5 + {}^-4 =$ <u>1</u>

Multiple Choice

9. (a)

10. (b)

11. (c)

12. (a)

77

13. (a) 2

(b) 2

(c) <u>add</u> the <u>opposite</u>

14. (a) $4 - 3 = 1$
$4 + {}^{-}3 = 1$

(b) ${}^{-}3 - 2 = {}^{-}5$
${}^{-}3 + {}^{-}2 = {}^{-}5$

(c) $4 - {}^{-}3 = 7$
$4 + 3 = 7$

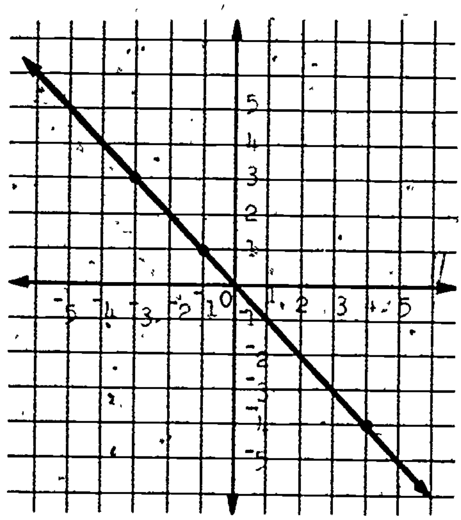
(d) ${}^{-}3 - {}^{-}2 = {}^{-}1$
${}^{-}3 + 2 = {}^{-}1$

Exercises - Page 13-1e.

1. (a) S.
(b) R
(c) T
(d) B

2.

$f : x \rightarrow$ opp x	
Input	Output
x	opp x
4	${}^{-}4$
${}^{-}3$	3
0	0
${}^{-}1$	1



3. (a) 3 <u><</u> 5

(b) 7 <u>></u> 2

(c) ${}^{-}6$ <u><</u> 4

(d) 5 <u>></u> ${}^{-}8$

(e) ${}^{-}4$ <u><</u> ${}^{-}3$

(f) ${}^{-}16$ <u><</u> ${}^{-}1$

4. (a) $3 + 5 = \underline{8}$

(b) $3 + {}^-5 = \underline{{}^-2}$

(c) $4 - 3 = \underline{1}$
$4 + {}^-3 = 1$

(d) $4 - {}^-3 = \underline{7}$
$4 + 3 = 7$

(e) $6 - {}^-12 = \underline{18}$
$6 + 12 = 18$

(f) $8 + {}^-7 = \underline{1}$

(g) ${}^-4 + 7 = \underline{3}$

(h) ${}^-8 - {}^-5 = \underline{{}^-3}$
${}^-8 + 5 = {}^-3$

(i) ${}^-4 - 7 = \underline{{}^-11}$
${}^-4 + {}^-7 = {}^-11$

(j) ${}^-3 - {}^-6 = \underline{3}$
${}^-3 + 6 = 3$

(k) $14 + {}^-14 = \underline{0}$

(ℓ) $7 - 7 = \underline{0}$
$7 + {}^-7 = 0$

(m) ${}^-9 + 9 = \underline{0}$

(n) ${}^-9 - {}^-9 = \underline{0}$
${}^-9 + 9 = 0$

Lesson 13-2.

Avoid using the expression "Two negatives make a positive" when discussing multiplication of integers. Students too often transfer this easily remembered saying to the addition operation. You may substitute a catch-phrase such as "Like signs, positive products; unlike signs, negative products."

Also avoid using the symbol "÷" to indicate division. The fraction bar is preferred.

Class Discussion - Page 13-2.

1. positive
2. negative
3. negative
4. positive

$\frac{{}^-12}{3} = \underline{{}^-4}$

$\frac{{}^-12}{{}^-3} = \underline{4}$

$\frac{12}{{}^-3} = \underline{{}^-4}$

5. positive

6. positive

7. negative

Exercises - Page 13-2b.

1. 105
2. 15
3. $^-16$
4. $^-8$
5. $^-27$
6. 9
7. 30
8. $^-6$
9. $^-45$
10. $^-6$
11. 56
12. 3
13. $^-4$
14. $^-95$
15. $^-4$
16. 112
17. 225
18. $^-25$
19. 30
20. $^-13$
21. $^-176$
22. $^-1150$
23. 171
24. $^-81$
25. 702

Lesson 13-3.

Briefly review the comparison property:

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \cdot d = b \cdot c \text{ .}$$

$$\frac{a}{b} < \frac{c}{d} \text{ if and only if } a \cdot d < b \cdot c \text{ .}$$

Students should realize the importance of writing fractions with (a) no negative sign if the number is positive and (b) the negative symbol in the numerator if the number is negative.

It is strongly recommended that the teacher make every effort to use the word "simplify" instead of "reduce". (We define "simplify" to mean writing an equivalent fraction so that the numerator and denominator are relatively prime.) Encourage students to write their answers in simplest form, but do not overemphasize simplification. Correct answers that are not simplified are much preferred to simplified incorrect answers.

Class Discussion - Page 13-3.

1. $\frac{16}{^-4} = \underline{^-4}$ and $\frac{^-16}{4} = \underline{^-4}$

   $\frac{16}{4} = \underline{4}$ so $^-(\frac{16}{4}) = \underline{^-4}$

   They are all names for $\underline{^-4}$.

2. (b) $\frac{^-9}{2}$ $\underline{\frac{9}{^-2}}$ $\underline{^-(\frac{9}{2})}$ (e) $\frac{8}{^-9}$ $\underline{\frac{^-8}{9}}$ $\underline{^-(\frac{8}{9})}$

   (c) $\frac{6}{^-3}$ $\underline{\frac{^-6}{3}}$ $\underline{^-(\frac{6}{3})}$ (f) $\frac{^-4}{5}$ $\underline{\frac{4}{^-5}}$ $\underline{^-(\frac{4}{5})}$

   (d) $^-(\frac{5}{6})$ $\underline{\frac{^-5}{6}}$ $\underline{\frac{5}{^-6}}$ (g) $^-(\frac{2}{3})$ $\underline{\frac{^-2}{3}}$ $\underline{\frac{2}{^-3}}$

3. (a) $\frac{1 + {^-1}}{4} = \frac{0}{4}$ (or 0)

   (b) $\frac{^-4 + 2}{5} = \frac{^-2}{5}$

   (c) $\frac{^-9 + {^-7}}{10} = \frac{^-16}{10}$ (or $\frac{^-8}{5}$)

4. $\frac{4}{2} = \underline{2}$ and $\frac{2}{^-1} = \underline{^-2}$

5. (a) 2

   (b) 2

6. $\frac{4}{2} = \underline{2}$ and $\frac{^-4}{^-1} = \underline{4}$

1. (a) $\frac{^-15}{13} < \frac{3}{^-4}$

(b) $\frac{^-7}{^-8} > \frac{6}{11}$

(c) $\frac{18}{^-5} < \frac{4}{^-3}$

(d) $\frac{^-9}{8} < \frac{2}{^-3}$

(e) $\frac{^-1}{^-3} > \frac{1}{4}$

2. (a) $\frac{^-8}{8} = {}^-1$

(b) $\frac{^-1}{5}$

(c) $\frac{8}{6} = \frac{4}{3}$

Lesson 13-4.

The study of multiples was postponed until now in the hope that students would be less likely to confuse factor and multiple. Mention factors as little as possible.

Class Discussion - Page 13-4.

1. 2
2. 64, 66, 68, 70, 72, 74, 76, 78, 80
3. 800, 20,000, 6,000,000,000
4. 0, 3, 6, 9, 12, 15, 18, 21, 24, 27
5. 0, 7, 14, 21, 28, 35, 42, 49, 56, 63
6. 0, 5, 10, 15, 20, 25, 30, 35, 40, 45
7. 0
8. 1
9. 2, 3, 4, 5, 16, 29

10. number
    multiple

11. 2 times

12. 1

13. 2 times
    1 and 3

14. Answers will vary.

Exercises - Page 13-4b.

1. 31 33 37 39 41 43 47 49 51 53 57 59
   61 63 67 69 71 73 77 79 81 83 87 89

2. $39 = 3 \times 13$
   $49 = 7 \times 7$
   $51 = 3 \times 17$
   $57 = 3 \times 19$
   $63 = 3 \times 21$ or 7×9
   $69 = 3 \times 23$
   $77 = 7 \times 11$
   $81 = 9 \times 9$ or 3×27
   $87 = 3 \times 29$

3. 0

4. (a) 24
   (b) 12
   (c) 8
   (d) 6
   (e) 4
   (f) 3
   (g) 2
   (h) 1

5. (a) 16
   (b) 8
   (c) 4
   (d) 2
   (e) 1

6. (a) 12
   (b) 6
   (c) 4
   (d) 3
   (e) 2
   (f) 1

7. (a) 15
   (b) 5
   (c) 3
   (d) 1

Lesson 13-5.

Students should not be asked to find the least common multiple by the prime factorization method. However, if some of them do use that method, don't discourage them. More important than finding the <u>least</u> common multiple is the ability to find <u>a</u> common multiple. Be sure students understand that a common multiple of two numbers can always be obtained by finding their product.

Class Discussion - Page 13-5.

1. (a) 2

   (b) 0, 2, 4, 6, or 8

   (c) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

   (Students may, of course, list 0. If they do, remind them of the fact that 0 is not usually included in a list of multiples.)

2. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

3. 6, 12, and 18

   The smallest is 6.

   There is no greatest common multiple of 2 and 3.

4. 24 is a common multiple of 3 and 4.

   The smallest is 12.

   The L.C.M. is 12.

5.

9	12
(9 + 9) = 18	24 = (12 + 12)
(9 + 18) = 27	36 = (12 + 24)
(9 + 27) = 36	

   The L.C.M. is 36. All are multiples of 36.

Exercises - Page 13-5b.

1. (a)

10	15
20	30
30	

   L.C.M. is 30.

(b)

8	10
16	20
24	30
32	40
40	

L.C.M. is 40.

(c)

15	20
30	40
45	60
60	

L.C.M. is 60.

(d)

9	15
18	30
27	45
36	
45	

L.C.M. is 45.

(e)

12	8
24	16
–	24

L.C.M. is 24.

(f)

6	9
12	18
18	

L.C.M. is 18.

(g)

21	14
42	28
	42

L.C.M. is 42.

2. (a) 30, The next three are 60, 90, and 120.
   (b) 40
   (c) 45, 90, 135

Class Discussion - Page 13-5d.

1. $4 < 8$ so you add 2 to 4.
   $6 < 8$ so you add 2 to 6.
2. 8, 8.
3. 12, 12
4. 20, 4

5. 24 , 24 , 8

6. 5 7
10 14
15 21
20 28
25 35
30
35

35 , 35

7. 6 (And 2 × 3 = 6)
10 (And 2 × 5 = 10)
14 (And 2 × 7 = 14)
15 (And 3 × 5 = 15)
21 (And 3 × 7 = 21)

prime numbers

8. 9 4
18 8
27 12
36 16
20
24
28
32
36

36 , 36

Exercises - Page 13-5f.

1. 10

2. 12

3. 12

4. 72

5. 72

6. 18

7. 90

8. 63

9. 48

10. 120

11. 33

12. 42

Lesson 13-6.

Allow students to use the flow chart for adding rational numbers as long as they need it. On the other hand, if some have developed other successful techniques, let them use the one they prefer.

Class Discussion - Page 13-6.

1. (a) $\frac{2 + 3}{8} = \frac{5}{8}$

(b) $\frac{2 + 1}{3} = \frac{3}{3} = 1$

(c) $\frac{5 + {}^{-}1}{8} = \frac{4}{8}$

$= \frac{1}{2}$

2. P(blue) $= \underline{\frac{2}{4}}$

$\frac{2 + 1}{4} = \frac{3}{4}$

P(either red or blue) $= \underline{\frac{3}{4}}$

3. Output: $\frac{3}{8}$, $\frac{3}{4}$, $\underline{\frac{36}{32}}$

$\frac{3}{8} \cdot \frac{4}{4} = \frac{12}{32}$ and $\frac{3}{4} \cdot \frac{8}{8} = \frac{24}{32}$

$\frac{12 + 24}{32} = \frac{36}{32}$

$\frac{36}{32} = \frac{9}{8}$

4. 8 is a <u>multiple</u> of 4 .

~~L.C.M.~~ of 8 and 4 is 8 .

$\frac{3}{4} = \frac{6}{8}$

$\frac{3+6}{8} = \frac{9}{8}$

Yes

5. (a) $\frac{15}{18}$

(b) No

(c) 6

$\frac{2}{3} = \frac{4}{6}$

(d) $\frac{4+1}{6} = \frac{5}{6}$

(e) Yes

6. (a) $\frac{17}{12}$

(b) Yes

(c) 12

7. (a) $\frac{27}{18}$

(b) $\frac{3}{2}$

(c) $\frac{4+5}{6} = \frac{9}{6}$

(d) No

(e) $\frac{3}{2}$

Exercises - Page 13-6e.

1. $\frac{5}{6}$
2. $\frac{19}{12}$
3. $\frac{13}{20}$
4. $\frac{4}{3}$
5. $\frac{5}{4}$
6. $\frac{23}{12}$
7. $\frac{3}{16}$
8. $\frac{2}{9}$
9. $\frac{37}{40}$
10. $\frac{11}{16}$
11. $\frac{7}{12}$
12. $\frac{13}{12}$
13. $\frac{1}{12}$
14. $\frac{9}{8}$
15. $\frac{19}{16}$
16. $\frac{^-3}{8}$
17. $\frac{1}{2}$
18. $\frac{^-1}{30}$
19. $\frac{3}{2}$
20. $\frac{15}{4}$

Lesson 13-7.

Exercises - Page 13-7.

1. $\frac{1}{8}$

$\frac{5}{8} + \frac{^-1}{2} = \frac{1}{8}$

2. $\frac{5}{12}$

$\frac{3}{4} + \frac{^-1}{3} = \frac{5}{12}$

3. $\frac{^-1}{3}$

$\frac{1}{2} + \frac{^-5}{6} = \frac{^-1}{3}$

4. $\frac{1}{2}$

$\frac{2}{3} + \frac{^-1}{6} = \frac{1}{2}$

5. $\frac{-3}{8}$

$\frac{1}{8} + \frac{-1}{2} = \frac{-3}{8}$

6. $\frac{-1}{12}$

$\frac{1}{4} + \frac{-1}{3} = \frac{-1}{12}$

7. $\frac{3}{16}$

$\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

8. $\frac{9}{16}$

$\frac{5}{8} + \frac{-1}{16} = \frac{9}{16}$

9. $\frac{-1}{6}$

$\frac{2}{3} + \frac{-5}{6} = \frac{-1}{6}$

10. $\frac{1}{6}$

$\frac{5}{6} + \frac{-2}{3} = \frac{1}{6}$

11. $\frac{11}{8}$

$\frac{7}{8} + \frac{1}{2} = \frac{11}{8}$

12. $\frac{1}{8}$

$\frac{7}{8} + \frac{-3}{4} = \frac{1}{8}$

Lesson 13-8.

The intent of this section is not to promote mastery in computations with exponents but rather to reinforce concepts of relationships within the decimal system.

The completed chart will show:

10,000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Class Discussion - Page 13-8.

1. (a) $\frac{1}{10}$

   (b) $\frac{1}{100}$

   (c) $\frac{1}{1000}$

2. (a) 0

   (b) $^{-}1$

   (c) $^{-}2$

   (d) $^{-}3$

3. $10^5 \times 10^3 = \underline{10^8}$

   $\frac{10^5}{10^2} = \underline{10^3}$

   $\frac{100,000,000}{\frac{1}{1000}} = \underline{10^{11}}$

Exercises - Page 13-8b.

1. (a) $10^{3+2} = 10^5$ and $1000 \times 100 = 100,000$

   $10^5 = 100,000$

   (b) $10^{5+^{-}3} = 10^2$ and $100,000 \times \frac{1}{1000} = 100$

   $10^2 = 100$

   (c) $10^{^{-}2+4} = 10^2$ and $\frac{1}{100} \times 10,000 = 100$

   $10^2 = 100$

   (d) $10^{^{-}3+^{-}2} = 10^{^{-}5}$ and $\frac{1}{1000} \times \frac{1}{100} = \frac{1}{100,000}$

   $10^{^{-}5} = \frac{1}{100,000}$

   (e) $10^{5+^{-}4} = 10^1$ and $100,000 \times \frac{1}{10,000} = 10$

   $10^1 = 10$

   (f) $10^{^{-}5+^{-}4} = 10^{^{-}9}$ and $\frac{1}{100,000} \times \frac{1}{10,000} = \frac{1}{1,000,000,000}$

   $10^{^{-}9} = \frac{1}{1,000,000,000}$

2. (a) $\frac{10^{7}}{10^{-2}} = 10^{9}$

(b) $\frac{10^{-2}}{10^{-4}} = 10^{2}$

(c) $10^{-3} \times 10^{4} = 10^{1}$

(d) $\frac{10^{-4}}{10^{1}} = 10^{-5}$

(e) $\frac{10^{-5}}{10^{-1}} = 10^{-4}$

Lesson 13-9.

The principal purpose here is to relate decimals and powers of 10.

Class Discussion - Page 13-9.

$10^{-2} = .01$

The completed chart will show:

Power of 10	Decimal Numeral	Fraction
10^{6}	1,000,000.	
10^{5}	100,000.	
10^{4}	10,000.	
10^{3}	1000.	
10^{2}	100.	
10^{1}	10.	
10^{0}	1.	
10^{-1}	.1	$\frac{1}{10}$
10^{-2}	.01	$\frac{1}{100}$
10^{-3}	.001	$\frac{1}{1000}$
10^{-4}	.0001	$\frac{1}{10,000}$

$10,000 \times \frac{1}{1000} = \frac{100,000}{1000} = 100$

$10^{5+^-3} = \underline{10^2}$

$10^6 \times 10^6 = \underline{10^{12}}$

$10^{^-5} \times 10^{^-6} = \underline{10^{^-11}}$

$\frac{10^2}{10^{^-4}} = \underline{10^6}$

Exercises - Page 13-9c.

1. (a) $10^4 \times 10^{^-4} = 10^0$
   (b) $10^2 \times 10^5 = 10^7$
   (c) $10^{^-3} \times 10^{^-2} = 10^{^-5}$
   (d) $10^{^-5} \times 10^{^-7} = 10^{^-12}$
   (e) $10^{^-2} \times 10^3 = 10^1$
   (f) $\frac{10^{^-3}}{10^{^-4}} = 10^1$
   (g) $\frac{10^1}{10^{^-3}} = 10^4$
   (h) $\frac{10^{^-4}}{10^{^-2}} = 10^{^-2}$

2. (a) $100,000 \times 1000 = 100,000,000$
   (b) $\frac{1}{100} \times \frac{1}{1000} = \frac{1}{100,000}$
   (c) $1,000,000 \times \frac{1}{10,000} = 100$
   (d) $1 \times \frac{1}{100} = \frac{1}{100}$

Lesson 13-10.

With the flow charts, students can easily rewrite any numeral using scientific notation, or convert from scientific notation to the usual numeral. Students are not required to do any computation involving scientific notation.

Class Discussion - Page 13-10a.

1. 5.64
2. 9.664
3. 2.0723

Exercises - Page 13-10d.

1. (a) 4.35×10^3
   (b) 2.40735×10^3
   (c) 1.968×10^{-4}
   (d) 4.5735×10^{-2}
   (e) 5.748216×10^5
   (f) 4.57×10^8
   (g) 8.46×10^{-3}
   (h) 9.945276×10^7
   (i) 3.84093×10^1
   (j) 7.5369×10^{-1}

2. (a) .0000615
   (b) 1001.
   (c) 549.2
   (d) .00028875
   (e) 9,626,400.
   (f) .073485

Lesson 13-11.

For this lesson each student needs three straight pins. Scissors are also needed. We believe the students' experience with this lesson is more than worth the extra time and trouble it takes to supply these extra items.

Although students often give lip service to the idea that $\frac{1}{4} = 25\%$ and $.25 = \frac{25}{100}$, many youngsters are not so sure of the truth of what they say that they are willing to use these symbols interchangeably.

Pinning down the relationships among fractions, decimals, and percents, literally as well as figuratively, should convince even the most uncertain student.

Class Discussion - Page 13-11.

On line C, the unit is divided into <u>10</u> parts.
On line D, the unit is divided into <u>10</u> parts.
On line E, the unit is divided into <u>8</u> parts.

1. E: $\frac{2}{8}$

F: $\frac{1}{5}$ and $\frac{2}{5}$, closer to $\frac{1}{5}$

D: .2 and .3

C: $\frac{2}{10}$ and $\frac{3}{10}$

B: 20% and 30%. This corresponds to 25%.

A: $\frac{20}{100}$ and $\frac{30}{100}$.

2. A: $\frac{60}{100}$

B: 60%

C: $\frac{6}{10}$

D: .6

E: $\frac{4}{8}$ and $\frac{5}{8}$ and closer to $\frac{5}{8}$

3. A: halfway between $\frac{87}{100}$ and $\frac{88}{100}$

B: between 87% and 88%

C: between $\frac{8}{10}$ and $\frac{9}{10}$ and closer to $\frac{9}{10}$

D: between .8 and .9 and closer to .9

G: halfway between $\frac{3}{4}$ and 1

4. A: $\frac{33}{100}$ and $\frac{34}{100}$

B: 33% and 34%

C: $\frac{3}{10}$ and $\frac{4}{10}$.

D: .3 and .4

E: $\frac{2}{8}$ and $\frac{3}{8}$

F: $\frac{1}{5}$ and $\frac{2}{5}$

G: $\frac{1}{4}$ and $\frac{2}{4}$

Exercises - Page 13-11c.

1. E: $\frac{6}{8}$ B: 75% A: $\frac{75}{100}$

2. D: .4 C: $\frac{4}{10}$ B: 40% A: $\frac{40}{100}$

3. H: $\frac{1}{3}$ and $\frac{2}{3}$

   G: $\frac{2}{4}$ C: $\frac{5}{10}$ B: 50%

4. A: $\frac{70}{100}$ B: 70% D: .7 F: $\frac{3}{5}$ and $\frac{4}{5}$

Lesson 13-12.

Techniques for renaming rational numbers are important if students are to be capable of using alternate methods of solving problems. Further practice in computation is provided in Chapter 17.

Class Discussion - Page 13-12.

1. The denominator is 100.

$$75\% = \frac{75}{100}$$

$$20\% = \frac{20}{100}$$

$$62\% = \frac{62}{100}$$

$$\frac{75}{100} = \frac{25}{25} \times \frac{3}{4} \text{ so } \frac{75}{100} = \frac{3}{4}$$

$$\frac{20}{100} = \frac{20}{20} \times \frac{1}{5} \text{ so } \frac{20}{100} = \frac{1}{5}$$

$$\frac{62}{100} = \frac{2}{2} \times \frac{31}{50} \text{ so } \frac{62}{100} = \frac{31}{50}$$

2. $30\% = \frac{30}{100}$

$150\% = \frac{150}{100}$

$5\% = \frac{5}{100}$

$\frac{30}{100} = .30$ (or .3)

$\frac{150}{100} = 1.50$ (or 1.5)

$\frac{5}{100} = .05$

3. $1\frac{1}{2} = 1.5$

To divide by 100, move the decimal point <u>2</u> places to the <u>left</u>, so 1.5 divided by 100 = <u>.015</u>. Therefore $1\frac{1}{2}\% =$ <u>.015</u>.

$.5\% = \frac{.5}{100}$

$.5\% = .005$

4. $\frac{1}{4} = \frac{25}{100}$

$\frac{25}{100} = 25\%$

$5 \times 100 = 8 \times \underline{62.5}$ $\quad 8\overline{)500.0}$ = 62.5 (or $62\frac{1}{2}$)

5. $.875 = \frac{875}{1000}$

$.875 \times 100 = 1000 \times \underline{87.5}$

Exercises - Page 13-12d.

1.

Fraction	Decimal	Percent
$\frac{1}{4}$	.25	25%
$\frac{1}{8}$	.125	$12\frac{1}{2}\%$
$\frac{3}{8}$	.375	$37\frac{1}{2}\%$
$\frac{13}{20}$	.65	65%
$\frac{4}{5}$	.8	80%
$\frac{63}{100}$	.63	63%
$\frac{2}{3}$	$.\bar{6}$	$66\frac{2}{3}\%$
$\frac{1}{8}$	.125	$12\frac{1}{2}\%$
$\frac{13}{10}$	1.3	130%
$\frac{9}{20}$	.45	45%
$\frac{1}{400}$	.0025	$\frac{1}{4}\%$
$\frac{11}{200}$	.055	5.5%
$\frac{1}{1000}$	.001	.1%

2. (a) 15

(b) $\frac{23}{40}$ or .575

(c) 120

(d) 27

(e) 60

(f) 100

(g) $\frac{7}{32}$ or .21875

(h) 78

(i) 552

(j) 110

(k) 24

Pre-Test Exercises - Page 13-P-1.

1. (a) 10

(b) 9
$13 + {}^{-}4 = 9$

(c) 2

(d) $^{-}2$

(e) $^{-}9$
$^{-}2 + {}^{-}7 = {}^{-}9$

(f) $^{-}3$
$3 + {}^{-}6 = {}^{-}3$

(g) 3

(h) 6
$4 + 2 = 6$

(i) 3
$^{-}6 + 9 = 3$

(j) $^{-}4$
$^{-}7 + 3 = {}^{-}4$

(k) $^{-}15$

(ℓ) $^{-}10$

(m) 0

2. (a) 24

(b) $^{-}63$

(c) 7

(d) $^{-}5$

(e) $^{-}32$

(f) 25

(g) $^{-}1$

(h) $^{-}3$

3. (a) $\frac{^-17}{3} \underline{<} \frac{14}{2}$

(b) $\frac{2}{^-6} \underline{>} \frac{^-5}{3}$

(c) $\frac{^-1}{^-4} \underline{<} \frac{2}{7}$

4. (a) $\frac{2}{6} = \frac{1}{3}$

(b) $\frac{4}{4} = 1$

5. (a) 0

(b) 1

(c) number itself

(d) multiple

(e) 4 , 8 , 12 , 16 , and 20

(f) even

(g) 2 , 3 , 5 , 7 , and 11

(h) 2

6. (a) greater (larger, bigger)

(b) 16

(c) product

(d) 35

(e) product

(f) 36

(g) 90

(h) 6

7. (a) $\frac{5}{6}$

(b) $\frac{17}{12}$

(c) $\frac{5}{8}$

(d) $\frac{^-5}{36}$

(e) $\frac{^-1}{12}$

8. (a) $\frac{1}{8}$

$\frac{3}{8} + {}^{-}\frac{1}{4} = \frac{1}{8}$

(b) $\frac{1}{12}$

$\frac{3}{4} + {}^{-}\frac{2}{3} = \frac{1}{12}$

(c) $\frac{7}{24}$

$\frac{9}{8} + {}^{-}\frac{5}{6} = \frac{7}{24}$

(d) $\frac{3}{10}$

$\frac{4}{5} + {}^{-}\frac{1}{2} = \frac{3}{10}$

(e) ${}^{-}\frac{1}{2}$

${}^{-}\frac{1}{3} + {}^{-}\frac{1}{6} = {}^{-}\frac{1}{2}$

9. (a) $10^{{}^{-}1+{}^{-}4} = 10^{{}^{-}5}$

(b) $10^{5-3} = 10^{2}$

(c) $10^{5+{}^{-}3} = 10^{2}$

(d) $10^{4-{}^{-}1} = 10^{5}$

(e) $\frac{10^{{}^{-}3}}{10^{{}^{-}2}} = 10^{{}^{-}1}$

(f) $10^{{}^{-}4} \times 10^{{}^{-}4} = 10^{{}^{-}8}$

10. (a) $10^{6} \times 10^{{}^{-}2} = 10^{4}$

(b) $10^{{}^{-}4} \times 10^{{}^{-}2} = 10^{{}^{-}6}$

(c) $10^{{}^{-}3} \times 10^{4} = 10^{1}$

(d) $\frac{10^{5}}{10^{{}^{-}4}} = 10^{9}$

11. (a) 4.56×10^{8}

(b) 2.97457×10^{2}

(c) $1.642 \times 10^{{}^{-}4}$

(d) 5.46007×10^{4}

(e) $8.7523 \times 10^{{}^{-}3}$

12. Fraction	Decimal Numeral	Percent
$\frac{7}{8}$	.875	87.5%
$\frac{1}{6}$	.16	$16\frac{2}{3}$%
$\frac{9}{10}$	.9	90%
$\frac{1}{200}$	.005	$\frac{1}{2}$%

Test - Page 13-T-1

1. (a) ⁻6
   (b) ⁻3
   ⁻4 + ⁻7 = ⁻3
   (c) 8
   (d) 7
   3 + 4 = 7
   (e) ⁻11
   ⁻9 + ⁻2 = ⁻11

2. (a) 27
   (b) 30
   (c) ⁻24
   (d) ⁻28

3. (a) ⁻9
   (b) 3
   (c) 1

4. (a) 12
   (b) 120
   (c) 45

5. (a) $\frac{11}{12}$

(b) $\frac{1}{2}$

(c) $\frac{31}{24}$

6. (a) $\frac{5}{12}$

$\frac{3}{4} + {}^{-}\frac{1}{3} = \frac{5}{12}$

(b) $\frac{1}{2}$

$\frac{5}{6} + {}^{-}\frac{1}{3} = \frac{1}{2}$

(c) ${}^{-}\frac{1}{8}$

$\frac{5}{8} + {}^{-}\frac{3}{4} = {}^{-}\frac{1}{8}$

7. (a) $10^3 \times 10^3 = 10^6$

(b) $10^{-3} \times 10^3 = 10^0$

(c) $\frac{10^3}{10^6} = 10^{-3}$

(d) $\frac{10^{-1}}{10^{-3}} = 10^2$

8. (a) 4.06×10^1

(b) 2.7482×10^8

(c) 6.821×10^{-5}

(d) 5.67×10^{-3}

9.

Fraction	Decimal Numeral	Percent
$\frac{1}{2}$	.5	50%
$\frac{2}{3}$	$.\bar{6}$	$66\frac{2}{3}$%
$\frac{9}{8}$	1.125	$112\frac{1}{2}$% or 112.5%
$\frac{1}{100}$	.01	1%
$\frac{1}{4}$	.25	25%
$\frac{1}{1000}$	.001	.1%

Teacher's Commentary

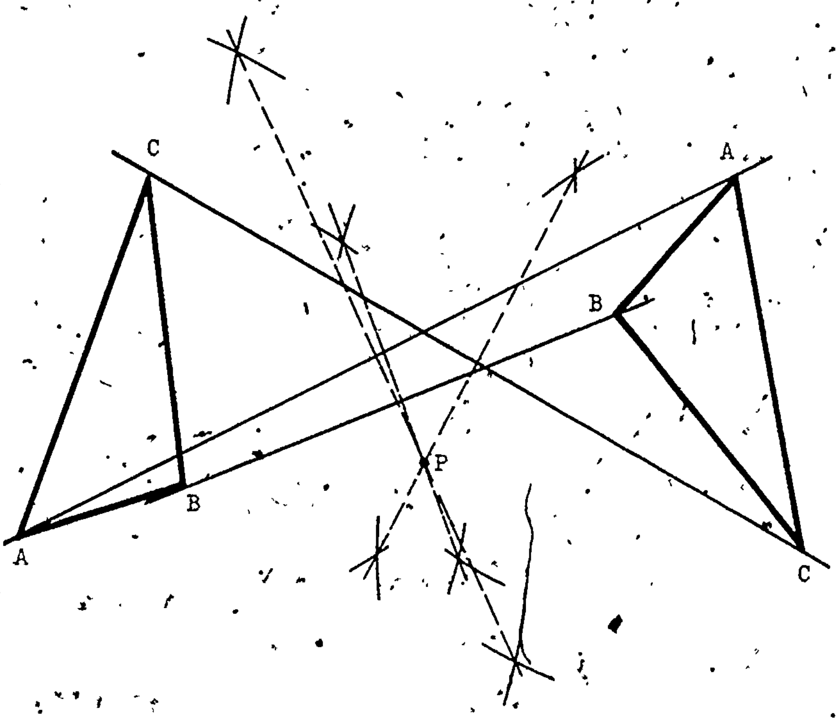
Chapter 14

PERPENDICULARS

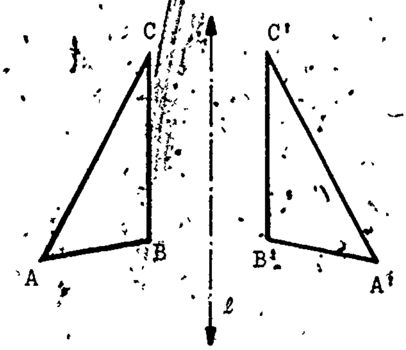
Lesson 14-1.

In Chapter 9 the students were introduced to the idea of congruence by tracing a figure and then seeing if one figure would fit upon a second figure. This process of moving a tracing sheet from one position to another suggests the idea of a rigid motion, mapping the plane onto itself. In this lesson we explore this idea more fully. In a practical sense, this is what people do when comparing two objects that can be physically manipulated. They in essence "slide" one object onto another, "turn" the object, or "flip" the object in order to determine whether there is a matching.

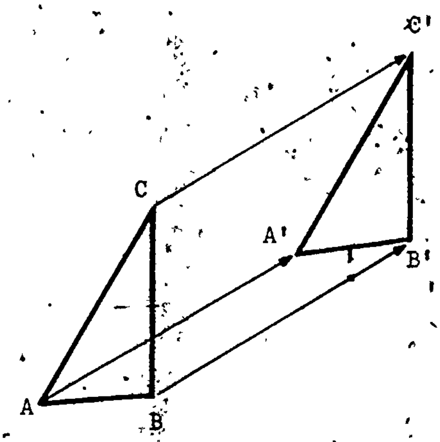
Although it is not obvious to the student, the motions of translation (slides), rotations (turns), and reflections (flips) are closely related to perpendicularity and parallelism. For example, given a figure in the plane and its image, the point about which the figure can be rotated such that it maps onto its image is the intersection of the perpendicular bisectors of the line segments between any two corresponding points on the figure and its image.



The motion of reflection is also related to perpendicularity. Given a figure and its reflection in a line ℓ, line ℓ is the perpendicular bisector of the line segments connecting corresponding points of the figure and its image. Line ℓ is the axis of symmetry.



It should be obvious how the motion of translation is related to parallelism.

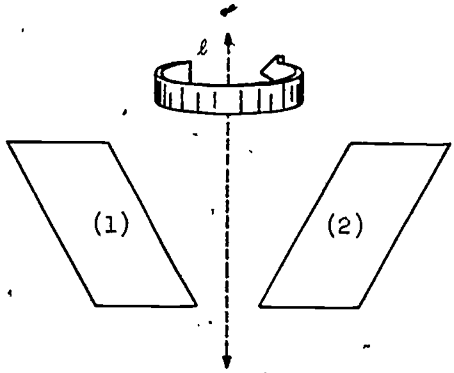


Although a primary purpose is to refine the student's geometric perception it is recommended that those students who have difficulty perceiving these motions should be allowed to make tracings of the figures and move the tracing from one position to another, flipping, turning, and sliding as the case may be.

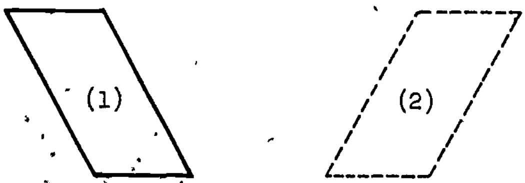
Exercises - Page 14-1c.

1. Here are some possible solutions for mapping the figures onto themselves. There are others.

   (a) Flip about ℓ.



Flip/Slide



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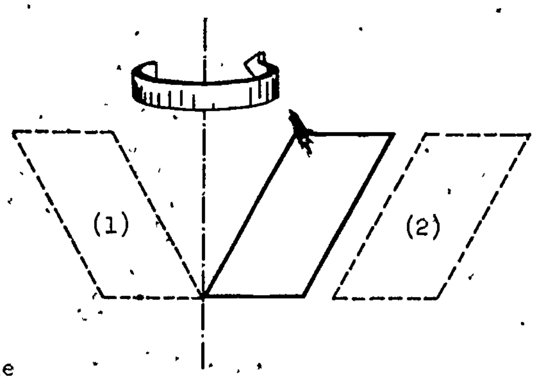

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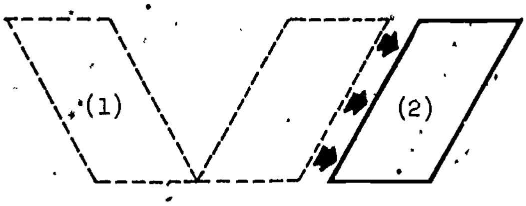
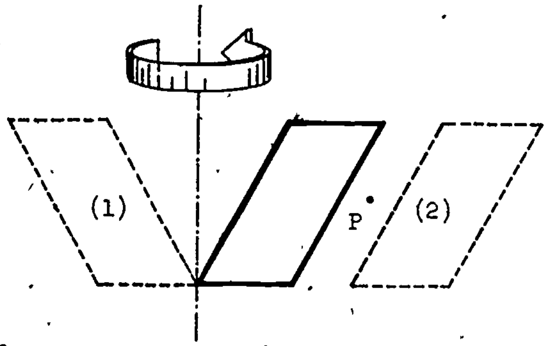
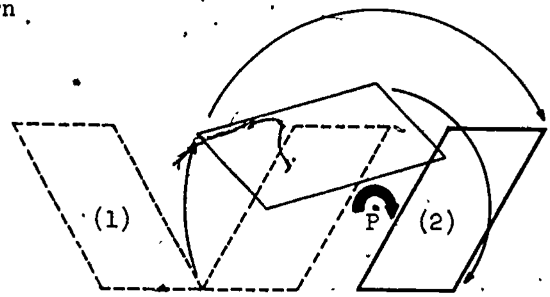


Flip/Turn

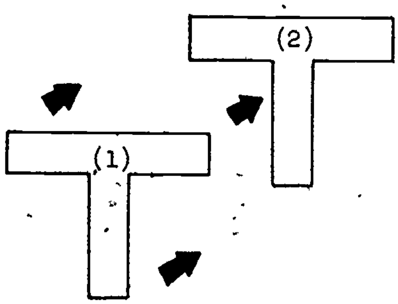
Flip



Turn

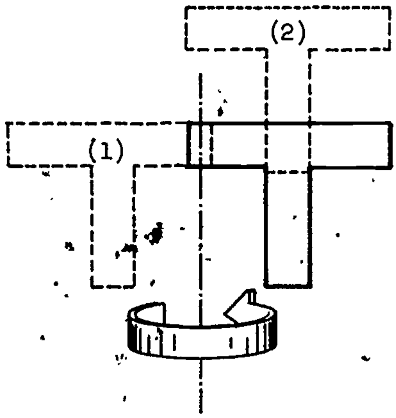


(b) Slide

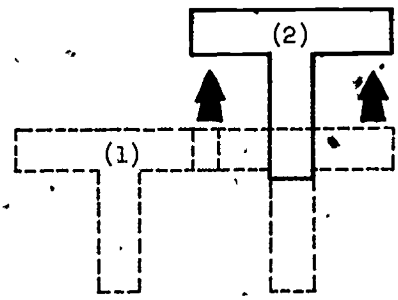


Flip/Slide

Flip



Slide



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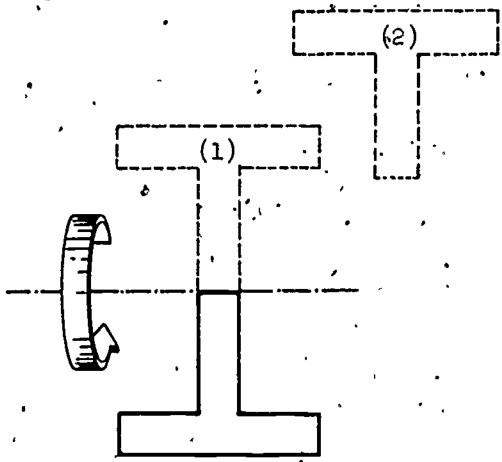
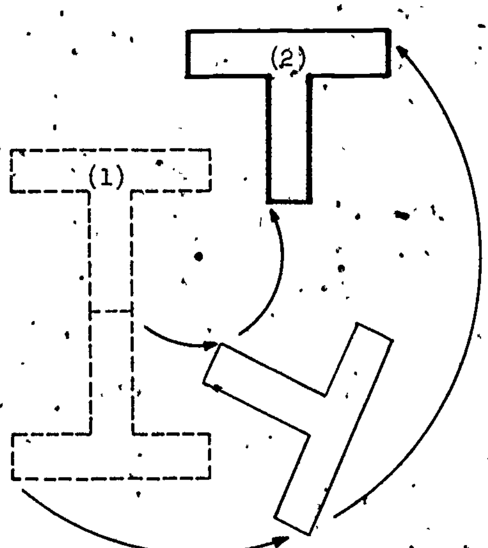
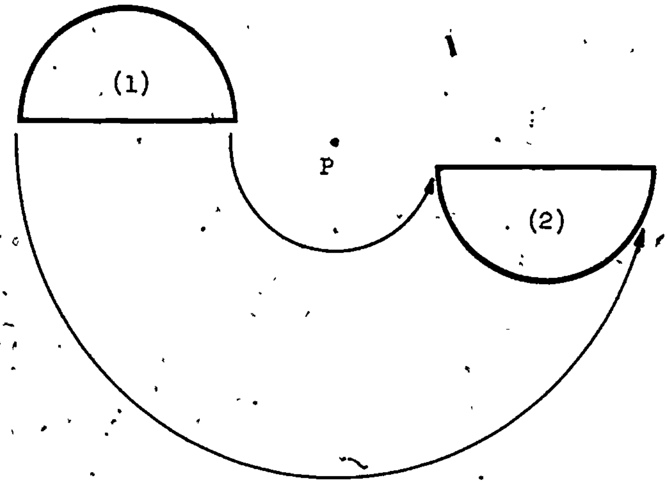


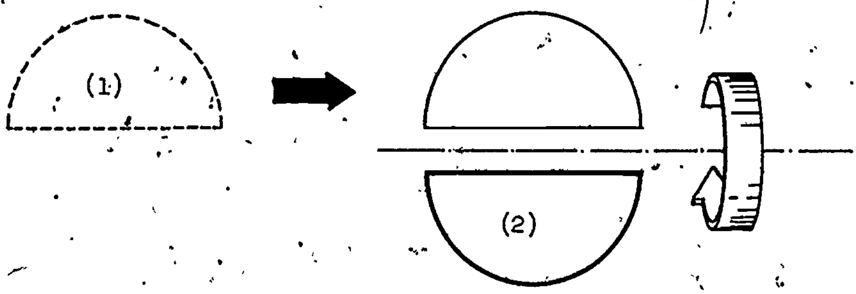
Turn



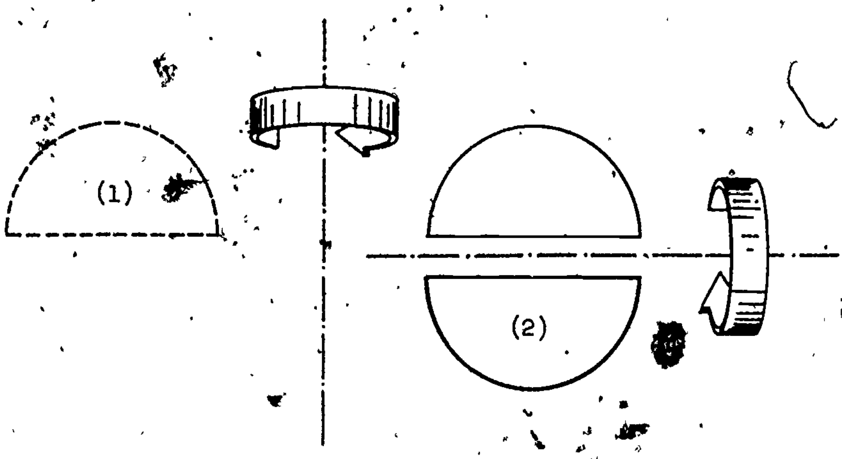
(c) Turn about P



Slide/Flip

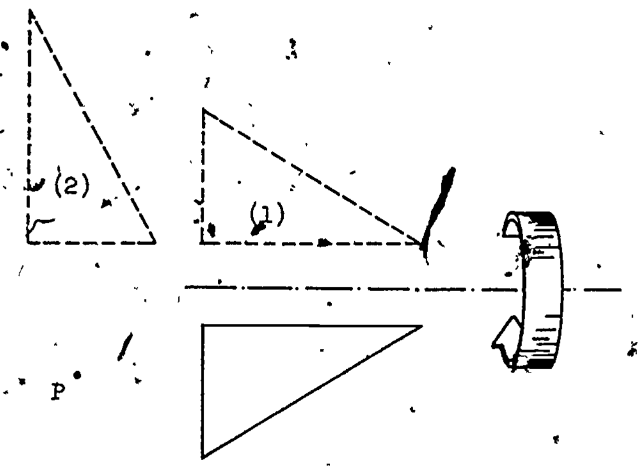


Flip/Flip

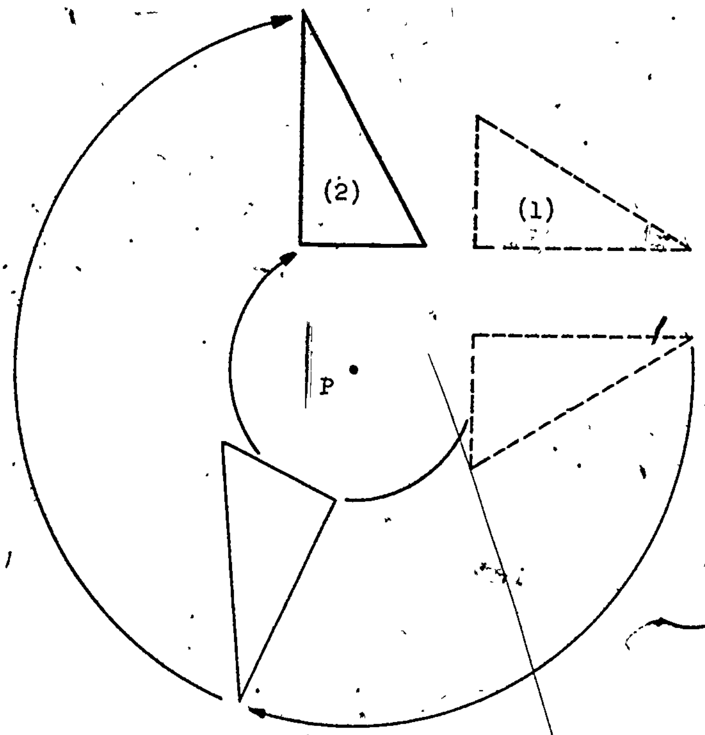


(d) Flip/Turn

Flip

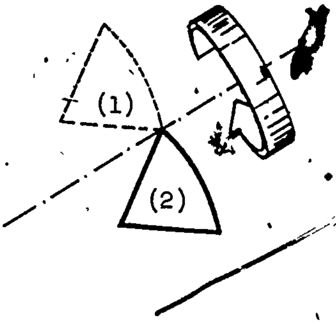


Turn

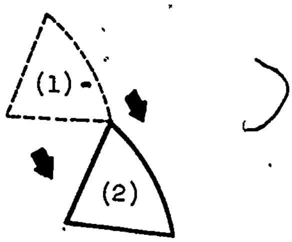


(e)

Flip

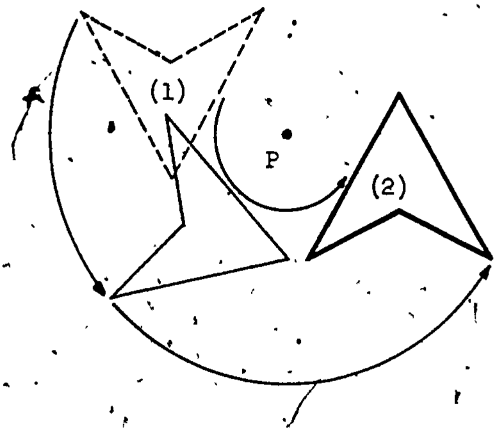


Slide

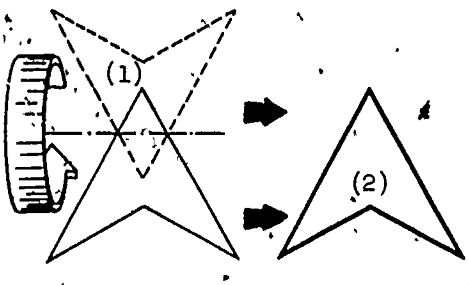


(f)

Turn

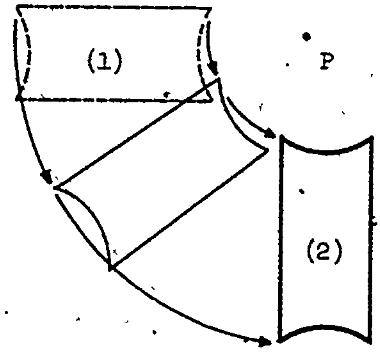


Flip/Slide



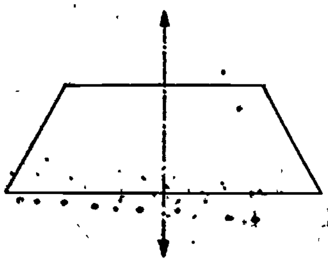
(g)

Turn

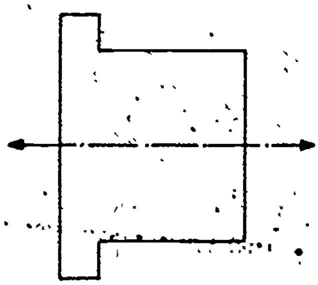


2. Figures (b), (d), and (e).

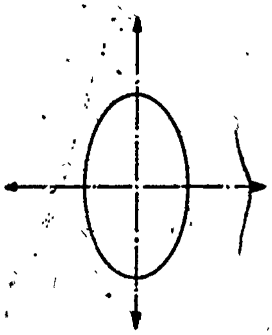
3. (a)



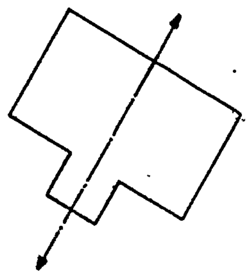
(b)



(c)



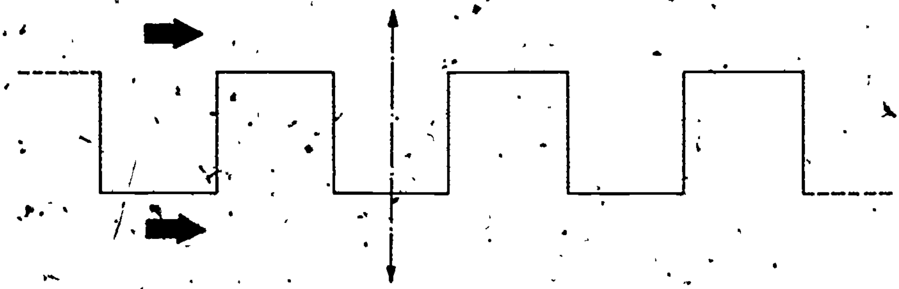
(d)



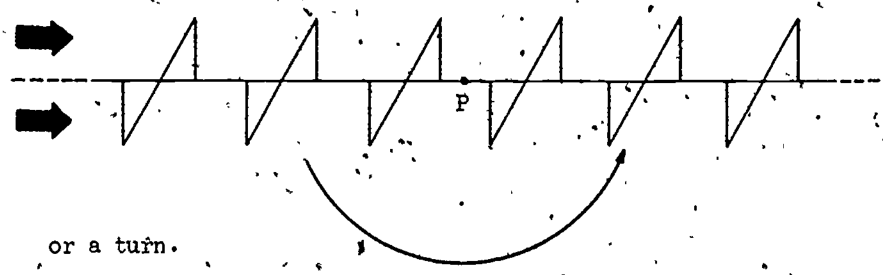
4. A circle.

5. BRAINBOOSTER.

(a) A slide or a flip.

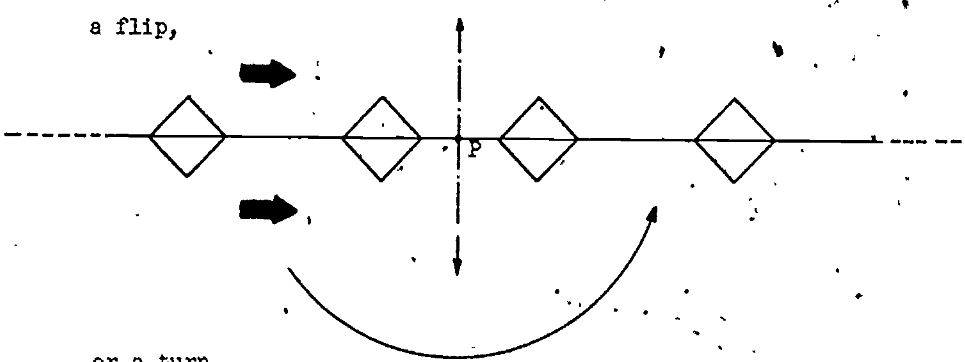


(b) A slide



or a turn.

(c) A slide,
a flip,



or a turn.

Lesson 14-2.

Since students have a tendency to forget that a figure is congruent to itself (the identity congruence) you should emphasize this idea.

Notice that you will have to provide a 3 X 5 card (construction paper will do) for each student.

Class Discussion - Page 14-2b.

1. (c) ABCD $\cong$ CDAB

2. (c) ABCD $\cong$ BADC

3. (c) ABCD $\cong$ DCBA

Exercises 14-2c.

1. (a) ABCD $\cong$ ABCD (identity congruence)

   (b) ABCD $\cong$ BADC (flip about the vertical axis)

2. (a) $\triangle$ ABC $\cong$ $\triangle$ ABC
   (b) $\triangle$ ABC $\cong$ $\triangle$ CAB
   (c) $\triangle$ ABC $\cong$ $\triangle$ BCA
   (d) $\triangle$ ABC $\cong$ $\triangle$ ACB
   (e) $\triangle$ ABC $\cong$ $\triangle$ BAC
   (f) $\triangle$ ABC $\cong$ $\triangle$ CBA

3. There are eight correspondences which can be found as follows. The identity correspondence followed by rotating the square clockwise through 90°, 180°, and 270° results in:

   RSTV $\cong$ RSTV

   RSTV $\cong$ VRST .

   RSTV $\cong$ TVRS

   RSTV $\cong$ STVR

   Flipping the square about the vertical axis results in

   RSTV $\cong$ SRVT .

   Flipping the square about the horizontal axis results in

   RSTV $\cong$ VTSR .

   Flipping the square about the diagonal through V and S results in

   RSTV $\cong$ TSRV .

Flipping the square about the diagonal through R and T results in

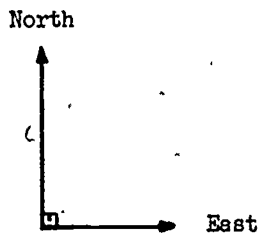
$$RSTV \cong RVTS .$$

Lesson 14-3.

Many students are not aware of the agreement that North is always placed at the top of a chart. You should point this out before assigning the exercises.

Exercises - Page 14-3.

1. (a)



(b) West

(c) South

2. $\overrightarrow{CB}$ and $\overrightarrow{CD}$

3. (a) $m \angle COB = 60$

(b) $m \angle AOB = 30$

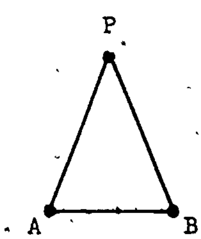
(c) $m \angle DOA = 120$

4. Fold and crease the paper once. Then fold the crease back on itself. The creases will be perpendicular to each other.

Lesson 14-4.

Class Discussion - Page 14-4.

1.



2. A triangle.

3. (a) m $\overline{PA}$ = m $\overline{PB}$

(b) Isosceles

4. (a) Yes

(b) Yes

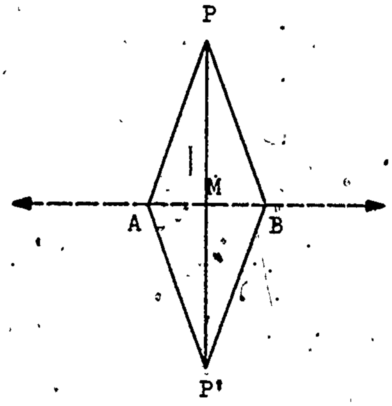
(c) Yes

(d) Yes

(e) Congruent

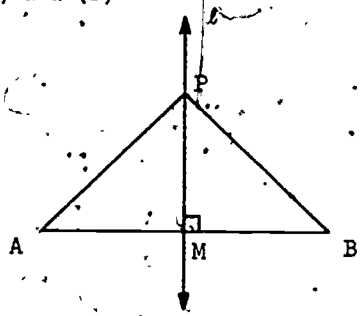
(f) A rhombus

5.



Exercises - Page 14-4a.

1. (a) and (b)



(c)

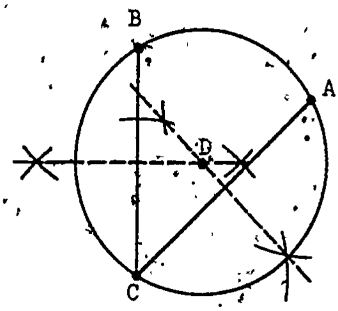
(1) $\overline{MA} \cong \overline{MB}$ because it is given that point M is the midpoint of $\overline{AB}$.

(2) $\angle AMP \cong \angle BMP$ because both angles are right angles.

(3) $\overline{MP} \cong \overline{MP}$ identity.

(4) $\triangle AEM \cong \triangle BPM$ by SAS.

2.

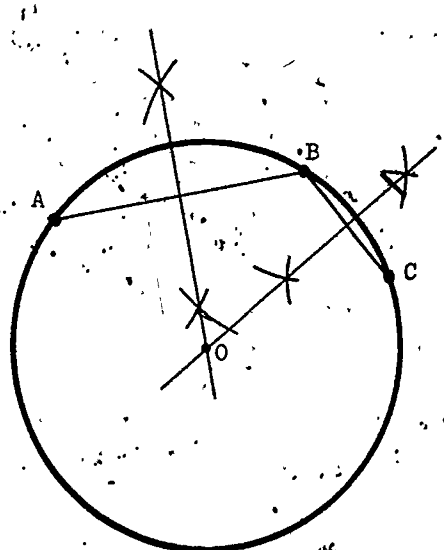


(d) Yes

(g) Yes

(h) Yes

3.

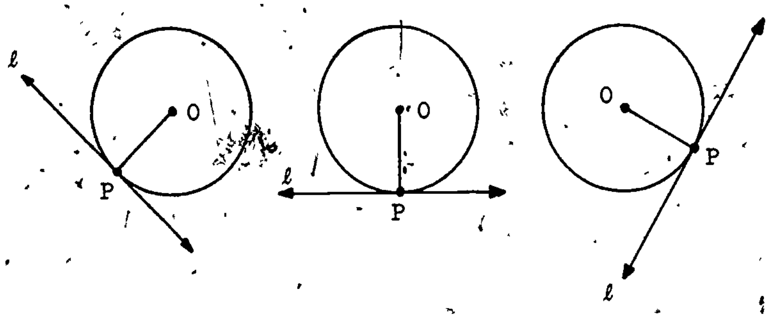


(c) A circle.

Lesson 14-5.

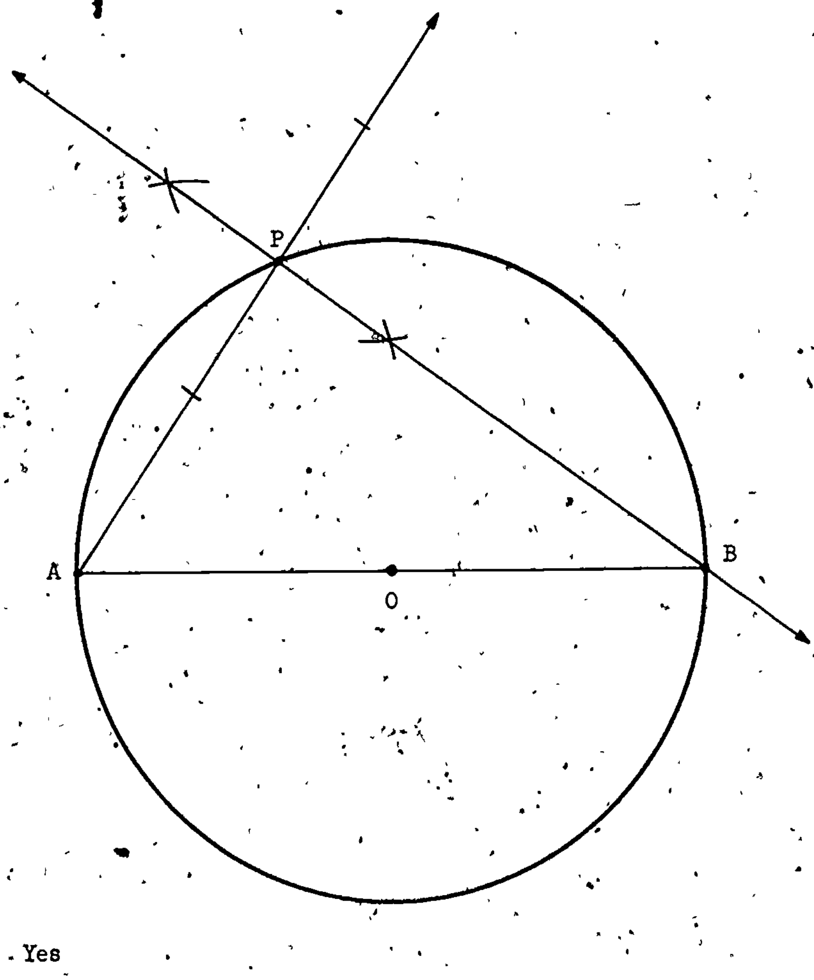
Class Discussion - Page 14-5.

1.



2. (a) They are perpendicular to each other.

3.



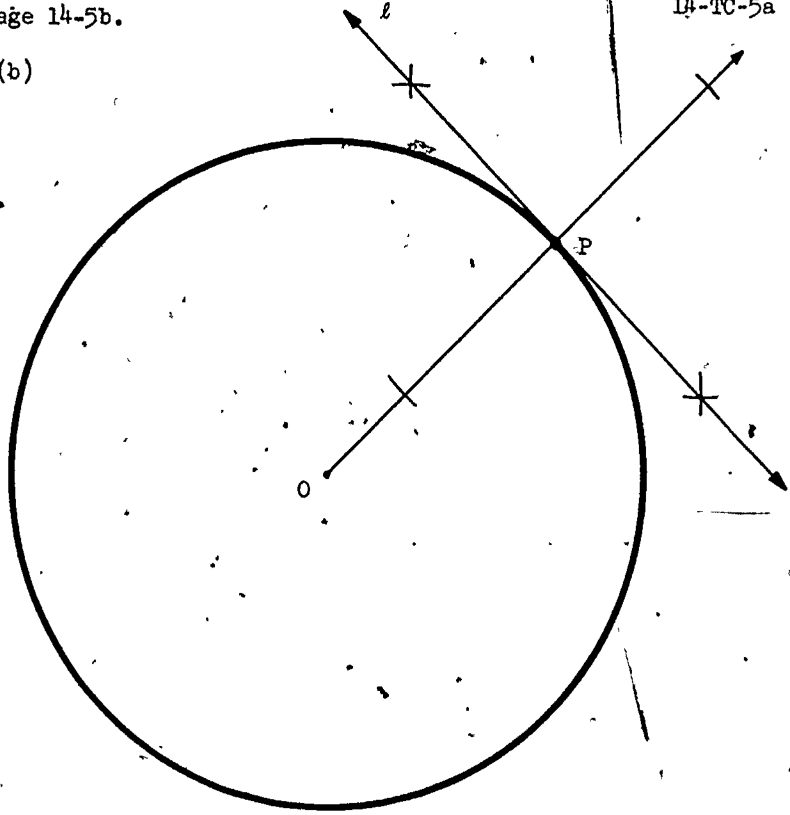
(c) Yes

(d) A right angle.

(e) A right triangle.

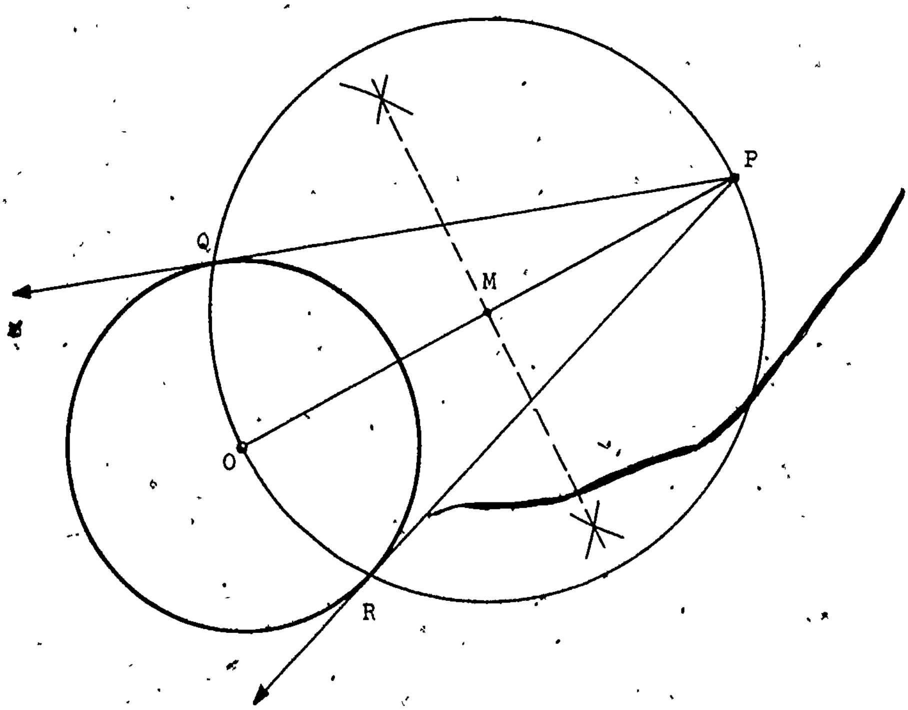
Exercises - Page 14-5b.

1. (a) and (b)



(c) Tangent

2.



3. BRAINBOOSTER. The path of point C will scribe a semi-circle.

Lesson 14-6.

Class Discussion - Page 14-6.

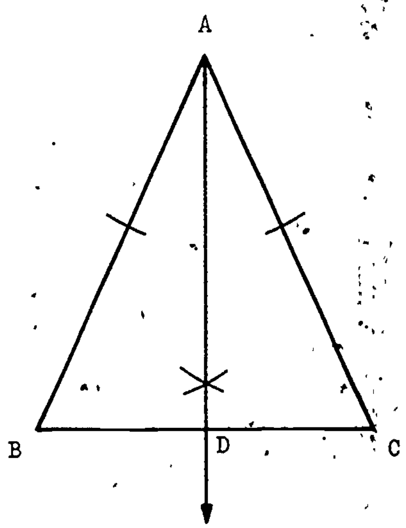
The answer to all questions is yes.

The paper folding in this section provides the students with first hand experiences in seeing that the medians, angle bisectors, and altitudes of a triangle do intersect. The construction of medians, angle bisectors, and altitudes with ruler and compass is difficult to carry out accurately enough to achieve the purpose of the constructions. Paper folding provides a quick and simple means of finding these lines.

Exercises - Page 14-6g.

1. (a) $\overline{DM}$
   (b) $\overline{CM}$
   (c) $\overline{CE}$
   (d) $\overline{CF}$

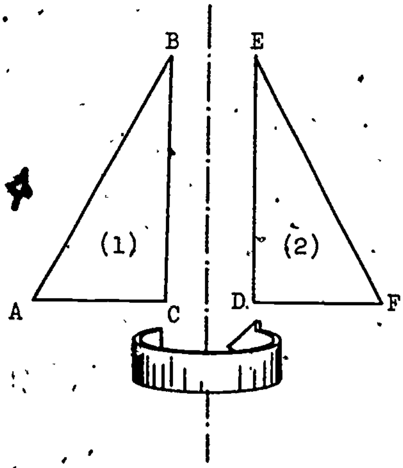
2.



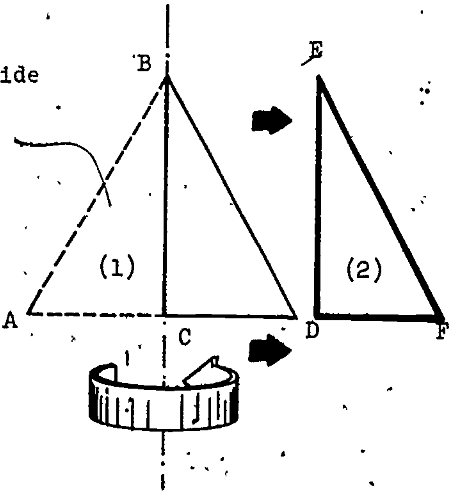
If the constructions are done carefully the bisector of $\overline{BC}$, the altitude from A to $\overline{BC}$, the median from A to $\overline{BC}$, and the bisector of $\angle A$ will all be on the same ray $\overrightarrow{AD}$.

Pre-Test - Page 14-P-1.

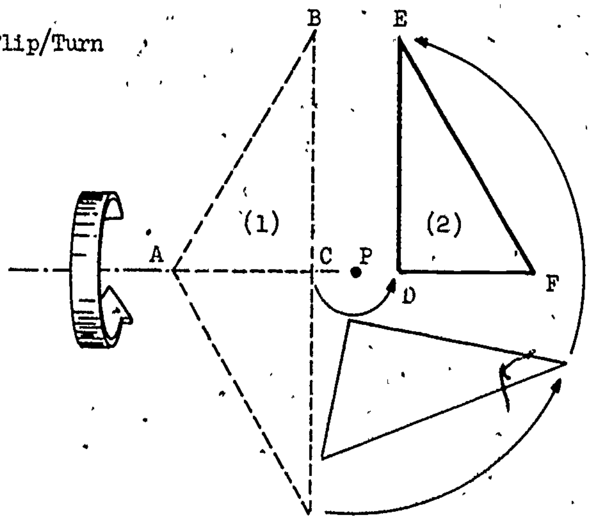
1. Flip



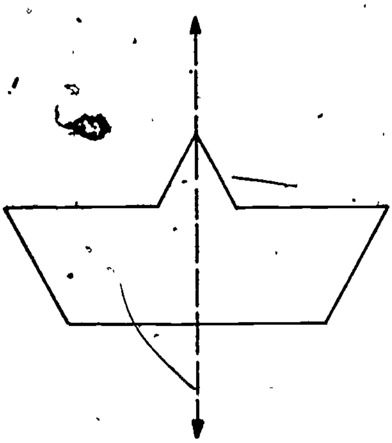
Flip/Slide



Flip/Turn



2.



3. An uncountable number of flip axes.

4. $\overline{CB}$ corresponds to $\overline{DE}$

$\overline{CA}$ corresponds to $\overline{DF}$

$\overline{BA}$ corresponds to $\overline{EF}$

$\angle c$ corresponds to $\angle d$

$\angle b$ corresponds to $\angle e$

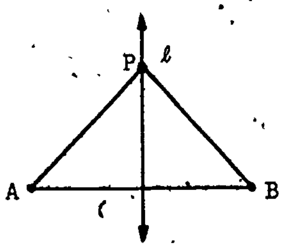
$\angle a$ corresponds to $\angle f$

5. (a) $\triangle ACB \cong \triangle ACB$

(b) $\triangle ACB \cong \triangle BCA$

6. $\overrightarrow{OA} \perp \overrightarrow{OC}$

7.



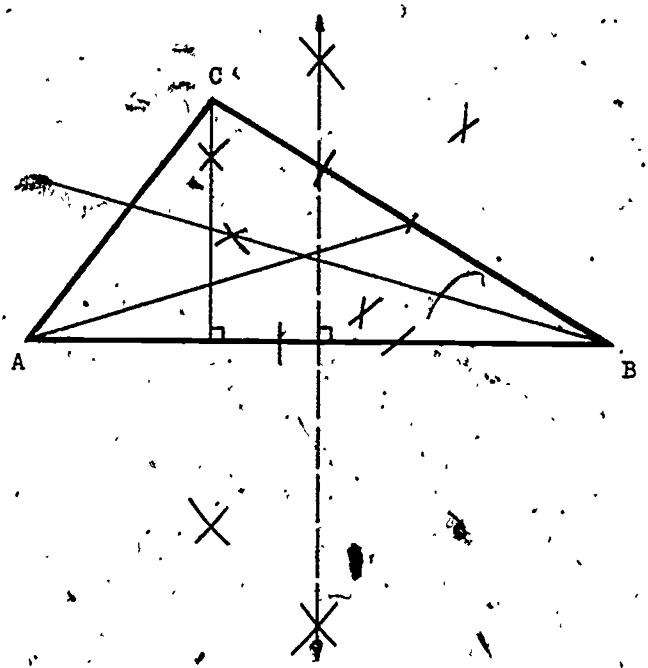
The triangle is an isosceles triangle.

8. Center

9. Tangent

10. Perpendicular

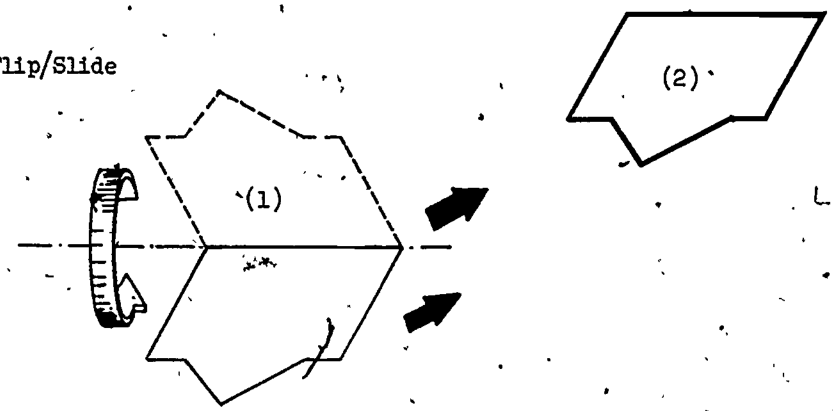
11.



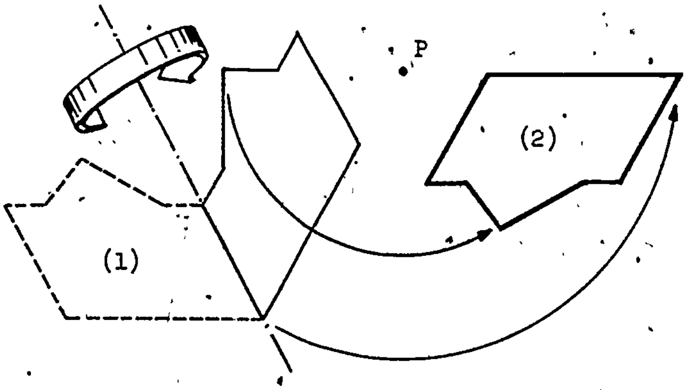
Test - Page 14-T-1.

1.

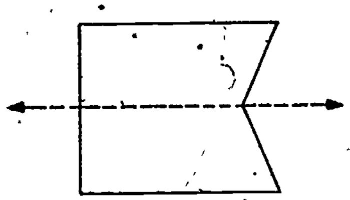
Flip/Slide



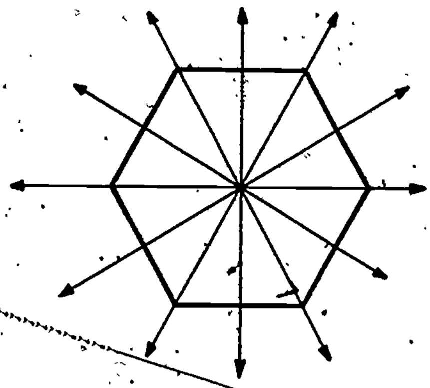
Flip/Turn



2.



3. Six



4. $\overline{AB}$ corresponds to $\overline{ED}$

$\overline{CB}$ corresponds to $\overline{FD}$

$\overline{AC}$ corresponds to $\overline{EF}$

$\angle a$ corresponds to $\angle e$

$\angle b$ corresponds to $\angle d$

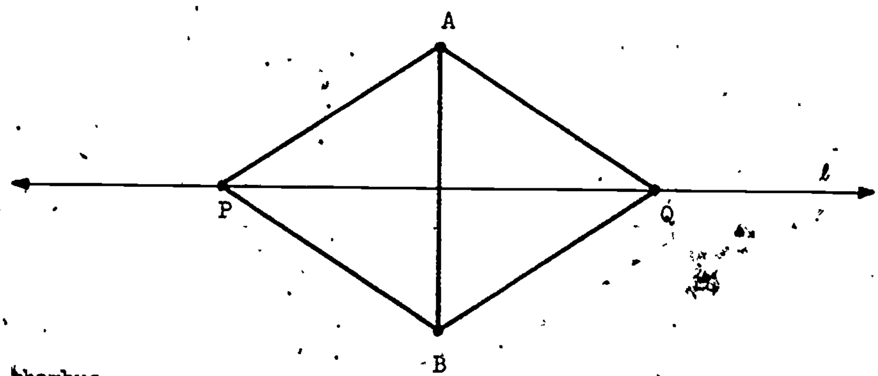
$\angle c$ corresponds to $\angle f$

5. (a) $\triangle ACB \cong \triangle ACB$

(b) $\triangle ACB \cong \triangle BCA$

6. $\overrightarrow{OA} \perp \overrightarrow{OC}$

7.



A rhombus.

8. Equidistant

9. One

10. Tangent

11. (a) Segment $\overline{DM}$ is called the perpendicular bisector of $\overline{AB}$.

    (b) Segment $\overline{CE}$ is called an altitude.

    (c) Segment $\overline{CM}$ is called a median.

    (d) Ray $\overrightarrow{BF}$ bisect the angle at B.

Teacher's Commentary

Chapter 15

MEASUREMENT

This chapter is divided into two general categories:

(1) physical measurement,

(2) exact geometric length.

Many students still have difficulty reading rulers. We hope that the student will gain this skill early in the chapter. We also hope that the student will recognize that all physical measurement is approximate.

In the last part of the chapter the concept is that we can imagine exact lengths of geometric figures even though exact physical measurement is impossible. It is through this concept that we are led to the need for some of the irrational numbers. These ideas introduce the next chapter, "Real Numbers".

Lesson 15-1.

Class Discussion - Page 15-1.

His rope is 20 yuks long.

His rope is 60 snuks long.

Exercises - Page 15-1a.

Be sure students know that they should use the zero mark on the ruler as their starting point, not the end of the ruler.

Segment	Ruler A	Ruler B	Ruler C	Ruler D
m $\overline{AB}$	4	10	6	14
m $\overline{AC}$	6	15	9	21
m $\overline{BC}$	7	18	11	25

Lesson 15-2.

Class Discussion - Page 15-2.

Segment $\overline{AB}$ is more than 2 inches long but less than 3 inches long.

Is the length of segment $\overline{AB}$ closer to 2 inches or 3 inches? 3 inches.

Is it exactly 3 inches long? No

Exercises - Page 15-2a.

1. (a) Each unit segment of ruler B is divided into 2 parts and each of these parts is $\frac{1}{2}$ inch long.

   (b) Each unit segment of ruler C is divided into 4 parts and each of these parts is $\frac{1}{4}$ inch long.

   (c) Each unit segment of ruler D is divided into 8 parts and each of these parts is $\frac{1}{8}$ inch long.

   (d) Each unit segment of ruler E is divided into 16 parts and each of these parts is $\frac{1}{16}$ inch long.

2. (a)

Ruler	Length of $\overline{CD}$
A	3
B	$2\frac{1}{2}$
C	$2\frac{3}{4}$
D	$2\frac{6}{8}$
E	$2\frac{12}{16}$

   (b) Which ruler is least precise? A

   (c) Which ruler is most precise? E

Lesson 15-3.

Class Discussion - Page 15-3.

1. $\frac{9}{4} = 2\frac{1}{4}$

2. $\frac{8}{5} = 1\frac{3}{5}$

3. $\frac{7}{6} = 1\frac{1}{6}$

4. $\frac{9}{2} = 4\frac{1}{2}$

Exercises - Page 15-3a.

1. $3\frac{3}{4} = 3 + \frac{3}{4}$

   $3\frac{3}{4} = \frac{12}{4} + \frac{3}{4}$

   $3\frac{3}{4} = \frac{15}{4}$

2. $4\frac{2}{3} = 4 + \frac{2}{3}$

   $4\frac{2}{3} = \frac{12}{3} + \frac{2}{3}$

   $4\frac{2}{3} = \frac{14}{3}$

3. $2\frac{5}{8} = 2 + \frac{5}{8}$

   $2\frac{5}{8} = \frac{16}{8} + \frac{5}{8}$

   $2\frac{5}{8} = \frac{21}{8}$

4. $5\frac{5}{9} = 5 + \frac{5}{9}$

   $5\frac{5}{9} = \frac{45}{9} + \frac{5}{9}$

   $5\frac{5}{9} = \frac{50}{9}$

Exercises - Page 15-3b.

1. $1\frac{5}{8} + 5\frac{1}{8} = 6\frac{6}{8}$

   $= 6\frac{3}{4}$

2. $7\frac{2}{9} + 8\frac{5}{9} = 15\frac{7}{9}$

3. $12\frac{2}{5} + 17\frac{2}{5} = 29\frac{4}{5}$

4. $15\frac{3}{10} + 9\frac{5}{10} = 24\frac{8}{10}$

   $= 24\frac{4}{5}$

Exercises - Page 15-3c.

1. $7\frac{5}{8} + 4\frac{3}{4} = 12\frac{3}{8}$

2. $5\frac{2}{3} + 12\frac{1}{2} = 18\frac{1}{6}$

3. $15\frac{7}{8} + 20\frac{5}{6} = 36\frac{17}{24}$

4. $8\frac{2}{7} + 1\frac{1}{2} = 9\frac{11}{14}$

Exercises - Page 15-3d.

1. $32\frac{2}{3}$
2. $56\frac{8}{9}$
3. $32\frac{2}{5}$
4. $43\frac{3}{4}$
5. $5\frac{6}{7}$
6. $14\frac{7}{8}$
7. 7
8. $12\frac{2}{9}$

Lesson 15-4.

Class Discussion - Page 15-4.

1. No
2. 2
3. Yes
4. $2\frac{3}{4}$

Exercises - Page 15-4a.

1. $m\,\overline{AB} \approx \underline{1\frac{1}{2}} - \underline{\frac{3}{4}}$

   $m\,\overline{AB} \approx \underline{\frac{3}{4}}$

2. $m\,\overline{AD} \approx \underline{3\frac{1}{2}} - \underline{\frac{3}{4}}$

   $m\,\overline{AD} \approx \underline{2\frac{3}{4}}$

3. $m\,\overline{BE} \approx \underline{5\frac{3}{4}} - \underline{1\frac{1}{2}}$

   $m\,\overline{BE} \approx \underline{4\frac{1}{4}}$

4. $m\,\overline{CE} \approx \underline{5\frac{3}{4}} - \underline{2\frac{3}{8}}$

   $m\,\overline{CE} \approx \underline{3\frac{3}{8}}$

5. $m\,\overline{BD} \approx \underline{3\frac{1}{2}} - \underline{1\frac{1}{2}}$

   $m\,\overline{BD} \approx \underline{2}$

6. $m\,\overline{CD} \approx \underline{3\frac{1}{2}} - \underline{2\frac{3}{8}}$

   $m\,\overline{CD} \approx \underline{1\frac{1}{8}}$

7. $m\,\overline{AC} \approx \underline{2\frac{3}{8}} - \underline{\frac{3}{4}}$

   $m\,\overline{AC} \approx \underline{1\frac{5}{8}}$

8. $m\,\overline{AE} \approx \underline{5\frac{3}{4}} - \underline{\frac{3}{4}}$

   $m\,\overline{AE} \approx \underline{5}$

9. $m\,\overline{DE} \approx \underline{5\frac{3}{4}} - \underline{3\frac{1}{2}}$

   $m\,\overline{DE} \approx \underline{2\frac{1}{4}}$

Class Discussion - Page 15-5.

1. (a) m $\overline{AB} \approx$ .9

m $\overline{BC} \approx$ .9

m $\overline{AC} \approx$ 1.3

The students may disagree on the measures of these segments. They may wish to argue as to which is the closest mark. Take advantage of such an opportunity to point out that the reading of any measuring instrument is subjective and often depends upon how the observer "sees" his instrument. In such cases either measurement is acceptable.

The perimeter of $\triangle ABC$ = m $\overline{AB}$ + m $\overline{BC}$ + m $\overline{AC}$

The perimeter of $\triangle ABC \approx$ .9 + .9 + 1.3

The perimeter of $\triangle ABC \approx$ 3.1

(b)

m $\overline{EF} \approx$ .7

m $\overline{FG} \approx$ 1.2

m $\overline{EG} \approx$ 1.4

The perimeter of $\triangle EFG \approx$ .7 + 1.2 + 1.4

The perimeter of $\triangle EFG \approx$ 3.3

(c) Each of these parts would be one one hundredth of an inch.

(d) Each part would be one one thousandth of an inch.
It would not be possible to read such a ruler.

Exercises - Page 15-5b.

1. m $\overline{AB} \approx$ 1.0
   m $\overline{BC} \approx$ 1.2
   m $\overline{AC} \approx$ 1.9

   P $\approx$ 4.1

2. m $\overline{EF} \approx$ 2.3
   m $\overline{FG} \approx$ .8
   m $\overline{EG} \approx$ 1.7

   P $\approx$ 4.8

3. m $\overline{JK} \approx$ 1.5
   m $\overline{KL} \approx$ 1.2
   m $\overline{JL} \approx$ 1.0

   P $\approx$ 3.7

Lesson 15-6.

Class Discussion - Page 15-6.

Each unit centimeter has been divided into 10 parts.

Can we write our measurements in decimal form? Yes

Segment	Length in inches	Length in cm
$\overline{AB}$	1	2.5
$\overline{CD}$	$2\frac{3}{4}$	7.0
$\overrightarrow{AC}$	$1\frac{7}{16}$	3.7

Without measuring, how long are $\overline{BC}$, $\overline{AD}$, and $\overline{BD}$? 3.7 cm

Without measuring, how long is $\overline{AO}$? 1.25 cm

Without measuring, how long is $\overline{DO}$? 3.5 cm

$\overline{AB}$ is 1 inch long and $\overline{AB}$ is 2.5 centimeters long. About how many centimeters are there in one inch?

2.5 cm ≈ 1 inch

Exercises - Page 15-6a.

1. m $\overline{AB}$ ≈ 4.1 cm
   m $\overline{BC}$ ≈ 1.4 cm
   m $\overline{AC}$ ≈ 4.4 cm

2. m $\overline{PQ}$ ≈ 2.9 cm
   m $\overline{QR}$ ≈ 4.7 cm
   m $\overline{QS}$ ≈ 3.4 cm
   m $\overline{PR}$ ≈ 7.1 cm

3. m $\overline{WX}$ ≈ 3.5 cm
   m $\overline{WZ}$ ≈ 4.4 cm
   m $\overline{XZ}$ ≈ 4.8 cm
   m $\overline{WY}$ ≈ 6.0 cm

4. m $\overline{EF}$ ≈ 1.8 cm
   m $\overline{FG}$ ≈ 4.7 cm
   m $\overline{FM}$ ≈ 3.0 cm
   m $\overline{GM}$ ≈ 3.0 cm

Lesson 15-7.

Lessons 15-7, 15-8, and 15-9 consider conversions of units within the British system and within the metric system.

We have taken a functional and graphical approach to conversion to reduce the confusion of whether to multiply by 12 or divide by 12, etc.

The graphical approach gives the student an easy method for converting units. We hope to establish the ideas of conversion through this visual approach. We are not attempting to develop a high degree of proficiency. Resist the temptation to supplement these sections with "drill" exercises.

Class Discussion - Page 15-7a.

To change from feet to inches you multiply by 12.

Write this function in arrow notation: $f : x \to 12x$.

Do you get 36 inches? Yes

2 feet = 24 inches

$3\frac{1}{2}$ feet = 42 inches

$1\frac{3}{4}$ feet = 21 inches

9 inches = $\frac{3}{4}$ feet

18 inches = $1\frac{1}{2}$ feet

27 inches = $2\frac{1}{4}$ feet

33 inches = $2\frac{3}{4}$ feet

48 inches = 4 feet

45 inches = $3\frac{3}{4}$ feet

3 inches = $\frac{1}{4}$ feet

You multiply the number of feet by 12 to change to number of inches.

You multiply the number of inches by $\frac{1}{12}$ to change to number of feet.

Exercises. - Page 15-7d.

1. (a) $2\frac{1}{4}$ feet = 27 inches

   (b) $3\frac{1}{2}$ feet = 42 inches

   (c) $\frac{3}{4}$ feet = 9 inches

   (d) $1\frac{1}{4}$ feet = 15 inches

   (e) 4 feet = 48 inches

2. (a) 18 inches = $\underline{1\frac{1}{2}}$ feet

(b) 45 inches = $\underline{3\frac{3}{4}}$ feet

(c) 33 inches = $\underline{2\frac{3}{4}}$ feet

(d) 21 inches = $\underline{1\frac{3}{4}}$ feet

(e) 15 inches = $\underline{1\frac{1}{4}}$ feet

Lesson 15-8.

Class Discussion - Page 15-8

1 yard = $\underline{1 \cdot 3}$ feet

2 yards = $\underline{2 \cdot 3}$ feet

3 yards = $\underline{3 \cdot 3}$ feet

4 yards = $\underline{4 \cdot 3}$ feet

5 yards = $\underline{5 \cdot 3}$ feet

x yards = $\underline{x \cdot 3}$ feet

$f : x \rightarrow \underline{3x}$

Exercises - Page 15-8b

1. (a) 4 yards = $\underline{12}$ feet

(b) $2\frac{1}{3}$ yards = $\underline{7}$ feet

(c) $7\frac{2}{3}$ yards = $\underline{23}$ feet

(d) 9 yards = $\underline{27}$ feet

(e) $\underline{9}$ yards = 27 feet

(f) $\underline{8}$ yards = 24 feet

(g) $\underline{5\frac{2}{3}}$ yards = 17 feet

(h) $\underline{6\frac{1}{3}}$ yards = 19 feet

(i) $\underline{2\frac{1}{3}}$ yards = 7 feet

(j) $\underline{8\frac{1}{3}}$ yards = 25 feet

(k) $\underline{\frac{2}{3}}$ yards = 2 feet

(l) $\underline{\frac{1}{3}}$ yards = 1 foot

2. (a) You multiply the number of yards by $\underline{3}$ to change to number of feet.

   (b) You multiply the number of feet by $\underline{\frac{1}{3}}$ to change to number of yards.

Class Discussion - Page 15-8c.

1. $1\frac{1}{3}$ yards = $\underline{4}$ feet = $\underline{48}$ inches.

2. 24 inches = $\underline{2}$ feet = $\frac{2}{3}$ yards.

   33 inches = $\underline{2\frac{3}{4}}$ feet = $\underline{\frac{11}{12}}$ yards.

The reason for this problem is to show the student that it is very difficult to read the graph to the required accuracy.

You multiply the number of inches by $\underline{\frac{1}{12}}$ to change to number of feet.

$$33 \text{ inches} = 33 \cdot \underline{\frac{1}{12}} \text{ feet}$$

You multiply the number of feet by $\underline{\frac{1}{3}}$ to change to number of yards.

$$33 \cdot \frac{1}{12} \text{ feet} = 33 \cdot \frac{1}{12} \cdot \underline{\frac{1}{3}} \text{ yards}$$

$33 \cdot \frac{1}{3} \cdot \frac{1}{12}$

$= \underline{11} \cdot \frac{1}{12}$

$= \frac{11}{12}$

Exercises - Page 15-8d.

2. 27 inches = $\frac{3}{4}$ yards

3. 45 inches = $1\frac{1}{4}$ yards

4. 60 inches = $1\frac{2}{3}$ yards

Lesson 15-9.

Before beginning this lesson draw on the chalkboard a segment more than 1.5 meters long. Have student mark off their 10 cm segments on this segment.

Class Discussion - Page 15-9a

1. 10

2. 10

3. 10

5. 1000

6. 10^{-3}

7. (a) 100

   (b) 10^2

8. (a) $\frac{1}{100}$

   (b) 10^{-2}

9. (a) 1000

   (b) $\frac{1}{1000}$

   (c) 10^{-3}

10. (a) right

    (b) left

1. (b) 438.6 mm
   (c) 829.1 mm
   (d) 142.3 mm
   (e) 317.1 mm
   (f) 20.5 mm

2. (b) 1.26 cm
   (c) .28 cm
   (d) 143.72 cm
   (e) 22.7 cm
   (f) .09 cm

3. (b) 341 cm
   (c) 820 cm
   (d) 36.1 cm
   (e) 1250 cm
   (f) 2.8 cm

4. (b) 2.37 m
   (c) 5.481 m
   (d) .47 m
   (e) 1.98 m
   (f) 3.36 m

5. (b) 7600 m
   (c) 650 m
   (d) 4270 m
   (e) 3800 m
   (f) 27,200 m

6. (b) 4.021 km
   (c) 1.9319 km
   (d) .2814 km
   (e) 8.7295 km
   (f) 10 km

7. BRAINBOOSTER.
   (a) 2300 mm
   (b) .14729 km
   (c) 1,000,000 mm
   (d) 43.028 m

Lesson 15-10.

Class Discussion - Page 15-10a.

1. 3
2. $4\frac{3}{4} \cdot 3 = 14\frac{1}{4}$ ft.
3. $\frac{1}{12}$.
4. $27 \cdot \frac{1}{12} = 2\frac{1}{4}$ ft.
5. $16\frac{1}{2}$

   The wire is 33 feet long.

6. 100

7. 434.34 cm

8. $\frac{1}{10}$

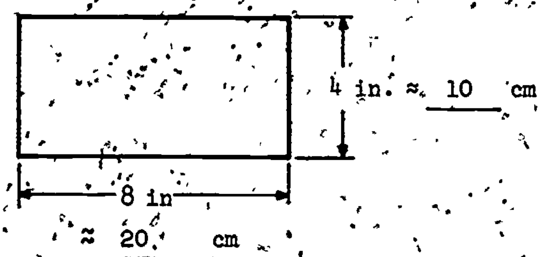
9. 68.58 cm

10. $\begin{array}{r} 434.34 \\ \underline{68.58} \\ 502.92 \end{array}$

11. $2 \cdot \underline{502.92} = 1005.84$ cm

Exercises - Page 15-10d.

1. (a)



(b) $P = 2(\ell + w)$

$P \approx 2(\underline{20} + \underline{10})$

$P \approx 2(\underline{30})$

$P \approx \underline{60}$ cm

2. (a) $m\,\overline{AB} \approx \underline{2}$ inches

$m\,\overline{AC} \approx \underline{4.8}$ inches

$m\,\overline{BC} \approx \underline{5.2}$ inches

(b) $P = m\,\overline{AB} + m\,\overline{AC} + m\,\overline{BC}$

$P \approx \underline{2} + \underline{4.8} + \underline{5.2}$

$P \approx \underline{12}$ inches

In all angle measurements permit an error of $\pm$ 2 degrees to allow for thickness of lines and inaccuracies in classroom protractors.

Class Discussion - Page 15-11b.

3. $m\angle x \approx$ 50 degrees.

Bottom of Page 15-11b.

2. No

3. $m\angle y \approx 130$ degrees

4. $m\angle x + m\angle y \approx 180$ degrees

Exercises - Page 15-11c.

1. (a) $m\angle a \approx$ 25
   $m\angle b \approx$ 155
   $\angle a$ and $\angle b$ are called supplementary.

   (b) $m\angle a \approx$ 45
   $m\angle b \approx$ 135
   $\angle a$ and $\angle b$ are called supplementary.

   (c) $m\angle a \approx$ 80
   $m\angle b \approx$ 100
   $\angle a$ and $\angle b$ are called supplementary.

2. (a) $m\angle p \approx$ 70
   (b) $m\angle s \approx$ 140
   (c) $m\angle t \approx$ 110
   (d) $\angle p$ and $\angle t$

3. (a) $m\angle x \approx$ 60
   (b) $m\angle y \approx$ 40
   (c) $m\angle z \approx$ 80

4. (a) m $\angle$ r ≈ 73

(b) m $\angle$ s ≈ 107 m $\angle$ q ≈ 73

m $\angle$ t ≈ 107 m $\angle$ u ≈ 73

m $\angle$ w ≈ 107 m $\angle$ v ≈ 73

m $\angle$ p ≈ 107

Lesson 15-12.

Class Discussion - Page 15-12.

1. (a) 12 feet

(b) 12 feet

2. (a) 5

(b) 9

(continued Class Discussion - Page 15-12b.)

1. 35 · 25

2. 25 · 15

3. 875 - 375

4. 500 sq. feet

If the tiles are each 1 square foot, how many do we need? 500

A rectangle with all its sides the same length is called a square.

A(square) = s^2

The area of the square below is 49 sq. inches.

1. $A = 14 \cdot 9$
   $A = 126$ sq. ft.

2. $A = 17^2$
   $A = 289$ sq mm

3. $A = 475 - 9$
   $A = 466$ sq cm

4. $A = 63 - 10$
   $A = 53$ sq. ft.

5. BRAINBOOSTER.
   Total Area $= a^2 + b^2 + 2ab$ square units

   or

   $(a + b)^2$ square units

Lesson 15-13.

Class Discussion - Page 15-13.

A 4-sided figure with opposite pairs of sides parallel is called a parallelogram.

1. $AB \parallel DC$
2. $\overline{BE}$
5. a rectangle
6. $\overline{AB}$
7. $\overline{AB}$
8. $\overline{BE}$
9. Yes

Exercises - Page 15-13b.

1. $A = 20 \cdot 13$
   $A = 260$ sq cm
2. $A = 9 \cdot 15$
   $A = 135$ sq. ft.
3. $A = 5 \cdot 13\frac{3}{4}$
   $A = 68\frac{3}{4}$ sq. in.

Class Discussion - Page 15-13c.

$\triangle$ WZY $\cong \triangle$ YXW

$A_{WXYZ} = b \cdot h$

$A_{\triangle WYZ} = \frac{1}{2} \cdot b \cdot h$

$A_{YXW} = \frac{1}{2} \cdot b \cdot h$

Exercises - Page 15-13d.

1\. (a) $A = \frac{1}{2} b h$

$A = \frac{1}{2} \cdot 13 \cdot 9$

$A = 58\frac{1}{2}$ sq. in.

(b) $A = \frac{1}{2} b h$

$A = \frac{1}{2} \cdot 4.6 \cdot 7.2$

$A = 16.56$ sq cm

(c) $A = \frac{1}{2} b h$

$A = \frac{1}{2} \cdot 5\frac{2}{3} \cdot 2\frac{1}{3}$

$A = 6\frac{11}{18}$ sq. yds.

2\. (a) $A_{(shaded)} = A_{(parallelogram)} - A_{(triangle)}$

$A_{(shaded)} = 432 - 216$

$A_{(shaded)} = 216$ sq. in.

(b) $A_{(shaded)} = A_{(rectangle)} + A_{(triangle)}$

$A_{(shaded)} = 101.22 + 21.84$

$A_{(shaded)} = 123.06$ sq cm

Lesson 15-14.

Class Discussion - Page 15-14b.

$m\,\overline{AB} \approx 5$

If we add the length of the sides we get 7.

No, this does not work.

If we multiply the length of the sides we get 12.

No, this does not work.

Does $(9 + 16) = 25$? Yes

Exercises - Page 15-14c.

1. (a) $12^2 + 5^2 = 144 + 25$

$12^2 + 5^2 = 169$

(b) $m\,\overline{KT} \approx 13$

$(m\,\overline{KT})^2 \approx 169$

(c) Yes

2. (a) $15^2 + 8^2 = 225 + 64$

$15^2 + 8^2 = 289$

(b) $m\,\overline{AE} \approx 17$

$(m\,\overline{AE})^2 \approx 289$

(c) Yes

3. (a) $24^2 + 10^2 = 576 + 100$

$24^2 + 10^2 = 676$

(b) $m\,\overline{GM} \approx 26$

$(m\,\overline{GM})^2 \approx 676$

(c) Yes

4. (a) $7.5^2 + 4^2 = \underline{56.25} + \underline{16}$

$7.5^2 + 4^2 = \underline{72.25}$

(b) $m\ \overline{PQ} \approx \underline{8.5}$

$(m\ \overline{PQ})^2 \approx \underline{72.25}$

(c) Yes

5. (a) $4.5^2 + 20^2 = \underline{20.25} + \underline{400}$

$4.5^2 + 20^2 = \underline{420.25}$

(b) $m\ \overline{RQ} \approx \underline{20.5}$

$(m\ \overline{RQ})^2 \approx \underline{420.25}$

(c) Yes

Pre-Test Exercises

1. (a) $\frac{7}{3} = \underline{2\frac{1}{3}}$

(b) $\frac{10}{7} = \underline{1\frac{3}{7}}$

(c) $\frac{17}{8} = \underline{2\frac{1}{8}}$

2. (a) $2\frac{1}{6} = \underline{\frac{13}{6}}$

(b) $3\frac{5}{8} = \underline{\frac{29}{8}}$

(c) $1\frac{3}{7} = \underline{\frac{10}{7}}$

3. (a) $1\frac{5}{8} + 5\frac{3}{8} = \underline{7}$

(b) $2\frac{4}{9} + 8\frac{3}{9} = \underline{10\frac{7}{9}}$

(c) $2\frac{2}{3} + 2\frac{2}{5} = \underline{5\frac{1}{15}}$

(d) $5\frac{1}{4} + 1\frac{1}{6} = \underline{6\frac{5}{12}}$

4. (a) $1\frac{1}{7} \times 5 =$ $5\frac{5}{7}$

(b) $3\frac{5}{8} \times 6 =$ $21\frac{3}{4}$

5. (a) m $\overline{AB} \approx$ $1\frac{3}{4}$

(b) m $\overline{AC} \approx$ $3\frac{3}{8}$

(c) m $\overline{BC} \approx$ $1\frac{5}{8}$

(d) m $\overline{BD} \approx$ $3\frac{3}{4}$

(e) m $\overline{CD} \approx$ $2\frac{1}{8}$

6. (a) $2\frac{1}{2}$ feet = 30 inches

(b) 21 inches = $1\frac{3}{4}$ feet

(c) 39 inches = $3\frac{1}{4}$ feet

(d) 15 inches $\approx$ 37.5 cm (exactly 38.10 cm)

(e) 17 feet = $5\frac{2}{3}$ yards

(f) $9\frac{2}{3}$ yards = 29 feet

(g) 48 inches = $1\frac{1}{3}$ yards

7. (a) 3.9 cm = 39 mm

(b) 400 cm = 4 m

(c) 2.6 m = 260 cm

(d) 427 mm = 42.7 cm

(e) 1.3 km = 1300 m

(f) 2938 mm = 2.938 m

8. (a) $m\angle a \approx$ 15 degrees

(b) $m\angle b \approx$ 125 degrees

(c) $m\angle c \approx$ 80 degrees

9. $\angle$ b and $\angle$ d are supplementary.

10. (a) A = 108 sq. ft.

(b) A = 105 sq. ft.

(c) A = 90 sq cm

Test

1. (a) $\frac{9}{7} = 1\frac{2}{7}$

(b) $\frac{14}{9} = 1\frac{5}{9}$

2. (a) $2\frac{1}{3} = \frac{7}{3}$

(b) $1\frac{3}{7} = \frac{10}{7}$

3. (a) $2\frac{3}{8} + 1\frac{7}{8} =$ $4\frac{1}{4}$

(b) $3\frac{2}{9} + 4\frac{5}{9} =$ $7\frac{7}{9}$

4. (a) $2\frac{2}{7} \times 2 = 4\frac{4}{7}$

(b) $3\frac{5}{9} \times 5 = 17\frac{7}{9}$

5. (a) $m\,\overline{AB} \approx$ $1\frac{3}{8}$

(b) $m\,\overline{AC} \approx$ $2\frac{3}{4}$

(c) $m\,\overline{BC} \approx$ $1\frac{3}{8}$

(d) $m\,\overline{CD} \approx$ $2\frac{3}{4}$

(e) $m\,\overline{BD} \approx$ $4\frac{1}{8}$

(f) $m\,\overline{AD} \approx$ $5\frac{1}{2}$

6. (a) $3\frac{3}{4}$ feet = 45 inches

(b) 33 inches = $2\frac{3}{4}$ feet

(c) 12 inches ≈ 30 cm (exactly 30.48 cm)

(d) $5\frac{2}{3}$ yards = 17 feet

(e) 19 feet = $6\frac{1}{3}$ yards

(f) 24 inches = $\frac{2}{3}$ yards

7. (a) 4.2 cm = 42 mm

(b) 525 cm = 5.25 m

(c) 1.3 m = 130 cm

(d) 7.3 km = 7300 m

8. (a) m $\angle$ a $\approx$ 45

(b) m $\angle$ b $\approx$ 155

(c) m $\angle$ c $\approx$ 25

(d) $\angle$ b and $\angle$ c are supplementary.

9. (a) A = 136 sq. ft.

(b) A = 207 sq. ft.

(c) A = $66\frac{1}{2}$ sq cm

Chapter 16

THE REAL NUMBERS

In this chapter we have taken a geometric approach to the development of irrational square roots. There are at least two reasons for this approach.

(1) Historically, the concept of irrational square roots came about from the consideration of the exact length of the diagonal of the square.

(2) We hope that a geometrical development will convince the students that these numbers do exist, and that it is possible to locate them on the number line.

In Chapter 17 we will take a brief look at the solutions of equations of the type $x^2 = c$, where c is any positive integer. This chapter provides the background for solving this equation.

Lesson 16-1.

Class Discussion - Page 16-1a.

1. $4^2 = \underline{16}$ $\quad 9^2 = \underline{81}$ $\quad 14^2 = \underline{196}$

$5^2 = \underline{25}$ $\quad 10^2 = \underline{100}$ $\quad 15^2 = \underline{225}$

$6^2 = \underline{36}$ $\quad 11^2 = \underline{121}$ $\quad 16^2 = \underline{256}$

$7^2 = \underline{49}$ $\quad 12^2 = \underline{144}$

$8^2 = \underline{64}$ $\quad 13^2 = \underline{169}$

2. $\sqrt{4} = 2$ $\quad \sqrt{49} = 7$

$\sqrt{9} = 3$ $\quad \sqrt{64} = 8$

$\sqrt{16} = 4$ $\quad \sqrt{81} = 9$

$\sqrt{25} = 5$ $\quad \sqrt{100} = 10$

$\sqrt{36} = 6$

3. (a) $\underline{\sqrt{9}}$

(b) $\underline{3}$

4. (a) $\sqrt{16}$

(b) 4

5. (a) $\sqrt{36}$

(b) 6

Exercises - Page 16-1e.

1. (a) $\sqrt{25}$

(b) 5

2. (a) $\sqrt{81}$

(b) 9

3. (a) $\sqrt{169}$

(b) 13

4. (a) $\sqrt{64}$

(b) 8

Lesson 16-2.

Class Discussion - Page 16-2.

1. (a) $3 = \sqrt{9}$

(b) $(\sqrt{9})^2 = 9$

2. (a) $5 = \sqrt{25}$

(b) $5^2 = 25$

$(\sqrt{25})^2 = 25$

3. (a) $6 = \sqrt{36}$

(b) $6^2 = 36$

$(\sqrt{36})^2 = 36$

4. (a) $7 = \sqrt{49}$

$7^2 = 49$

so $(\sqrt{49})^2 = 49$

5. (a) No

(b) No

6. (a) Yes

(b) No

(c) No

7. (a) Yes

(b) No

(c) No

8. (a) $\sqrt{3}$

(b) No

(c) $(\sqrt{3})^2 = 3$

9. (a) $\sqrt{5}$

(b) No

(c) $(\sqrt{5})^2 = 5$

10. (a) $\sqrt{6}$

(b) No

(c) $(\sqrt{6})^2 = 6$

11. (a) Yes
    (b) 8

12. (a) $(\sqrt{10})^2 = 10$
    (b) 10

Exercises - Page 16-2d.

1. (a) $\sqrt{2}$
   (b) $\sqrt{10}$
   (c) $\sqrt{18}$
   (d) $\sqrt{14}$

2. (a) 7
   (b) 12
   (c) 22
   (d) 45

3. (a) 5
   (b) 11
   (c) 9
   (d) 16
   (e) 55
   (f) 60

Lesson 16-3.

Class Discussion - Page 16-3b.

1.

Number	Write R if Rational or I if Irrational.	If Rational, write the integer name
$\sqrt{16}$	R	4
$\sqrt{5}$	I	
$\sqrt{7}$	I	
$\sqrt{8}$	I	
$\sqrt{36}$	R	6
$\sqrt{14}$	I	
$\sqrt{24}$	I	
$\sqrt{49}$	R	7
$\sqrt{40}$	I	
$\sqrt{81}$	R	9
$\sqrt{26}$	I	
$\sqrt{100}$	R	10
$\sqrt{64}$	R	8
$\sqrt{38}$	I	
$\sqrt{121}$	R	11
$\sqrt{18}$	I	
$\sqrt{25}$	R	5

2. (a) left
   (b) left
   (c) $<$
   (d) left
   (e) left
   (f) $<$
   (g) 2 and 3.

3. (a) left
   (b) left
   (c) <
   (d) 5
   (e) <
   (f) 4 and 5

4. (a) 3
   (b) 4
   (c) 3 and 4

5. 5 and 6

Exercises - Page 16-3d.

1.

Number	Write R if Rational or I if Irrational	*If* Rational write the integer name
$\sqrt{6}$	I	
$\sqrt{10}$	I	
$\sqrt{49}$	R	7
$\sqrt{60}$	I	
$\sqrt{121}$	R	11
$\sqrt{75}$	I	
$\sqrt{1}$	R	1
$\sqrt{50}$	I	

2. (a) 2 and 3
   (b) 3 and 4
   (c) 6 and 7
   (d) 1 and 2
   (e) 1 and 2

Class Discussion - Page 16-4.

....always opposite to the right angle.

$\overline{PR}$ is a units.

$\overline{PQ}$ is b units.

Example 1. For the right triangle above $a^2 =$ 36 and

$b^2 =$ 64.

$$a^2 + b^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

$$c = \sqrt{100}$$

So: $c = 10$

Example 2. For the right triangle above $a^2 =$ 4 and

$b^2 =$ 9

$$a^2 + b^2 = c^2$$

$$4 + 9 = c^2$$

$$13 = c^2$$

Exercises - Page 16-4b.

1. $a^2 + b^2 = c^2$

$\underline{1} + \underline{1} = c^2$

$\underline{2} = c^2$

$\underline{\sqrt{2}} = c$

2. $a^2 + b^2 = c^2$

$\underline{1} + \underline{4} = c^2$

$\underline{5} = c^2$

$\underline{\sqrt{5}} = c$

3. $a^2 + b^2 = c^2$

$\underline{9} + \underline{16} = c^2$

$\underline{25} = c^2$

$\underline{5} = c$

4. $a^2 + b^2 = c^2$

$\underline{1} + \underline{2} = c^2$

$\underline{3} = c^2$

$\underline{\sqrt{3}} = c$

5. $a^2 + b^2 = c^2$

$\underline{2} + \underline{3} = c^2$

$\underline{5} = c^2$

$\underline{\sqrt{5}} = c$

6. $a^2 + b^2 = c^2$

$\underline{1} + \underline{5} = c^2$

$\underline{6} = c^2$

$\underline{\sqrt{6}} = c$

Lesson 16-5.

Class Discussion - Page 16-5a.

We recommend that the teacher demonstrate these steps on the chalkboard as the students do their construction at their seats.

5. right

6. $\overline{BC}$

7. 1 unit

8. 1 unit

9. $(m\ \overline{BC})^2 = 1^2 + 1^2$

$(m\ \overline{BC})^2 = 2$

$m\ \overline{BC} = \sqrt{2}$

Page 16-5b.

3. $\underline{\sqrt{2}}$

4. $\underline{1}$

5. $(m\ \overline{BD})^2 = 1^2 + (\sqrt{2})^2$

$(m\ \overline{BD})^2 = \underline{\ 3\ }$

$m\ \overline{BD} = \underline{\sqrt{3}}$

page 16-5c.

3. $\underline{\sqrt{3}}$

4. $\underline{1}$

5. $(m\ \overline{BE})^2 = 1^2 + (\sqrt{3})^2$

$(m\ \overline{BE})^2 = \underline{\ 1\ } + \underline{\ 3\ }$

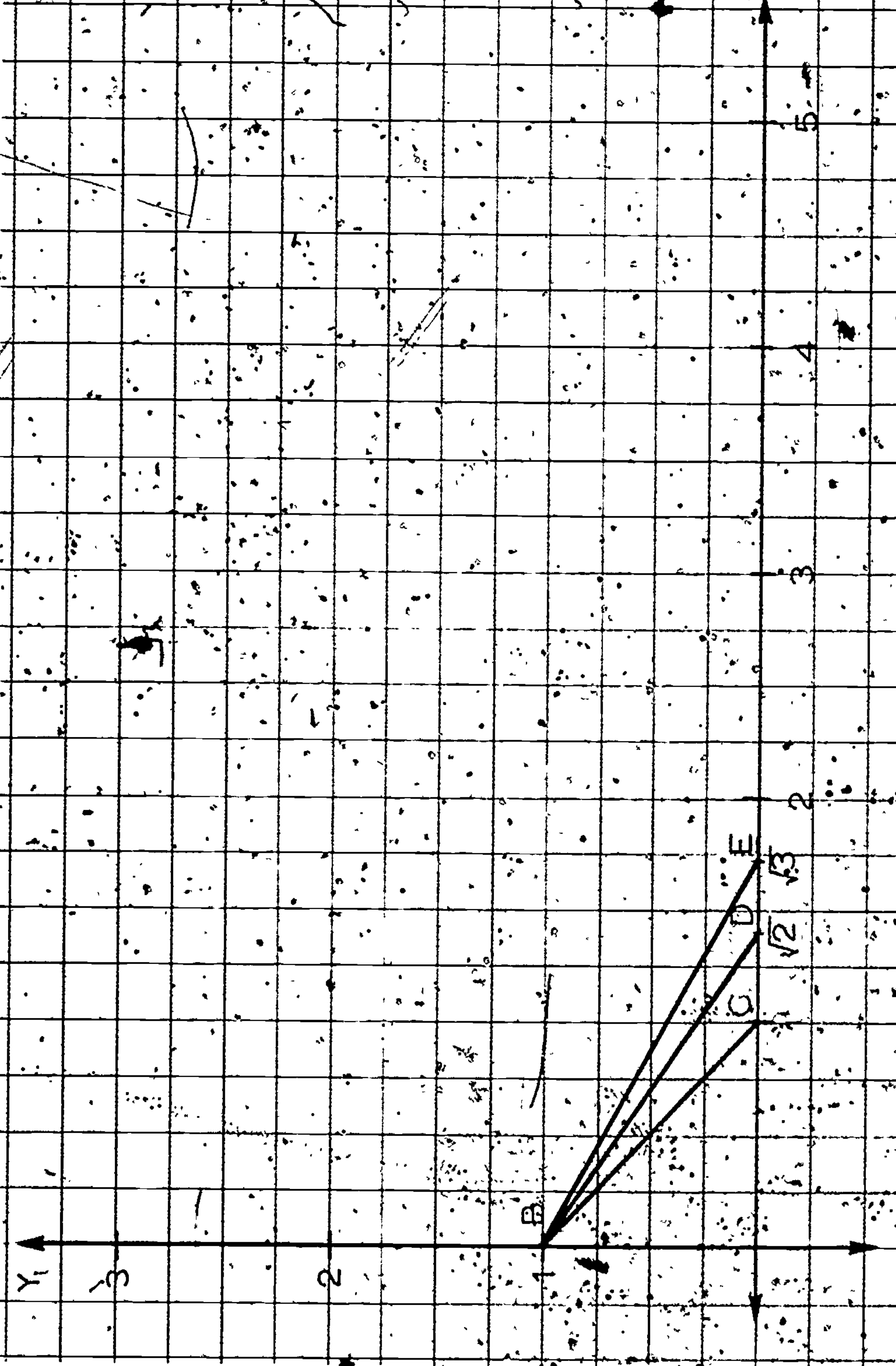
$(m\ \overline{BE})^2 = 4$

$m\ \overline{BE} = 2$

7. $\underline{2}$

WORKSHEET

Completed student construction



Exercises - Page 16-5f.

1. (a)
$$a^2 + b^2 = c^2$$
$$5^2 + 5^2 = c^2$$
$$50 = c^2$$
$$\sqrt{50} = c$$

(b)
$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = c^2$$
$$169 = c^2$$
$$13 = c$$

(c)
$$a^2 + b^2 = c^2$$
$$6^2 + 4^2 = c^2$$
$$52 = c^2$$
$$\sqrt{52} = c$$

2.
$$a^2 + b^2 = c^2$$
$$15^2 + 20^2 = c^2$$
$$625 = c^2$$
$$\sqrt{625} = c$$
$$25 = c$$

3.
$$a^2 + b^2 = c^2$$
$$10^2 + 24^2 = c^2$$
$$676 = c^2$$
$$\sqrt{676} = c$$
$$26 = c$$

Lesson 16-6.

Some students may have forgotten addition of integers using arrows on the number line. If so a review of this is appropriate at the beginning of this lesson.

Class Discussion - Page 16-6a.

Does $\sqrt{2} + \sqrt{5} = \sqrt{7}$? No

Is the point named? No

$\sqrt{2} + \sqrt{5}$ is between what two square roots? $\sqrt{13}$ and $\sqrt{14}$

Page 16-6b.

$\sqrt{2} + \sqrt{2} = \sqrt{8}$

$\sqrt{3} + \sqrt{3} = \sqrt{12}$

$\sqrt{5} + \sqrt{5} = \sqrt{20}$

$\sqrt{6} + \sqrt{6} = \sqrt{24}$

Do all of these come out to points that are named? Yes

$\sqrt{3} + \sqrt{3} = 2 \cdot \sqrt{3}$

$\sqrt{5} + \sqrt{5} = 2 \cdot \sqrt{5}$

$\sqrt{6} + \sqrt{6} = 2 \cdot \sqrt{6}$

Page 16-6d.

1. $\sqrt{7} \cdot \sqrt{11} = \sqrt{77}$
2. $\sqrt{5} \cdot \sqrt{7} = \sqrt{5 \cdot 7}$
   $= \sqrt{35}$
3. $\sqrt{6} \cdot \sqrt{8} = \sqrt{6 \cdot 8}$
   $= \sqrt{48}$

Exercises - Page 16-6d.

1. (a) No
   (b) Yes
   (c) No
   (d) Yes
   (e) Yes

2. (a) $\sqrt{24}$
   (b) $\sqrt{28}$
   (c) $\sqrt{27}$
   (d) $\sqrt{32}$
   (e) $\sqrt{20}$

3. (a) $\sqrt{14}$
   (b) $\sqrt{50}$
   (c) $\sqrt{60}$
   (d) $\sqrt{21}$
   (e) $\sqrt{24}$
   (f) $\sqrt{40}$
   (g) $\sqrt{56}$
   (h) $\sqrt{27}$

Lesson 16-7.

Treat this lesson primarily as exposure. Only a very modest amount of performance is expected.

Class Discussion - Page 16-7.

What is this name? 4

Page 16-7a.

$\sqrt{18} = \sqrt{9 \cdot 2}$

$\sqrt{18} = \sqrt{9} \cdot \sqrt{2}$

$\sqrt{18} = 3 \cdot \sqrt{2}$

Exercises - Page 16-7a.

1\. (a) $\sqrt{27} = \sqrt{\underline{9} \cdot 3}$

$\sqrt{27} = \underline{\sqrt{9}} \cdot \sqrt{3}$

$\sqrt{27} = \underline{3} \cdot \sqrt{3}$

(b) $\sqrt{28} = \sqrt{\underline{4} \cdot 7}$

$\sqrt{28} = \underline{\sqrt{4}} \cdot \sqrt{7}$

$\sqrt{28} = \underline{2} \cdot \sqrt{7}$

(c) $\sqrt{50} = \sqrt{\underline{25} \cdot 2}$

$\sqrt{50} = \underline{\sqrt{25}} \cdot \sqrt{2}$

$\sqrt{50} = \underline{5} \cdot \sqrt{2}$

(d) $\sqrt{45} = \sqrt{\underline{9} \cdot 5}$

$\sqrt{45} = \underline{\sqrt{9}} \cdot \sqrt{5}$

$\sqrt{45} = \underline{3} \cdot \sqrt{5}$

(e) $\sqrt{54} = \sqrt{\underline{9} \cdot 6}$

$\sqrt{54} = \underline{\sqrt{9}} \cdot \sqrt{6}$

$\sqrt{54} = \underline{3} \cdot \sqrt{6}$

(f) $\sqrt{40} = \sqrt{\underline{4} \cdot 10}$

$\sqrt{40} = \underline{\sqrt{4}} \cdot \sqrt{10}$

$\sqrt{40} = \underline{2} \cdot \sqrt{10}$

(g) $\sqrt{75} = \sqrt{\underline{25} \cdot 3}$

$\sqrt{75} = \underline{\sqrt{25}} \cdot \sqrt{3}$

$\sqrt{75} = \underline{5} \cdot \sqrt{3}$

(h) $\sqrt{48} = \sqrt{\underline{16} \cdot 3}$

$\sqrt{48} = \underline{\sqrt{16}} \cdot \sqrt{3}$

$\sqrt{48} = \underline{4} \cdot \sqrt{3}$

2\. (b) $\sqrt{6} \cdot \sqrt{7} = \sqrt{\underline{6} \cdot \underline{7}}$

$\sqrt{6} \cdot \sqrt{7} = \underline{\sqrt{42}}$

(c) $\sqrt{8} \cdot \sqrt{11} = \sqrt{\underline{8} \cdot \underline{11}}$

$\sqrt{8} \cdot \sqrt{11} = \underline{\sqrt{88}}$

$= \underline{2 \cdot \sqrt{22}}$

(d) $\sqrt{10} \cdot \sqrt{3} = \sqrt{\underline{10} \cdot \underline{3}}$

$\sqrt{10} \cdot \sqrt{3} = \underline{\sqrt{30}}$

(e) $\sqrt{2} \cdot \sqrt{8} = \sqrt{\underline{2} \cdot \underline{8}}$

$\sqrt{2} \cdot \sqrt{8} = \underline{\sqrt{16}}$

$= \underline{4}$

(f) $\sqrt{3} \cdot \sqrt{12} = \sqrt{\underline{3} \cdot \underline{12}}$

$= \underline{\sqrt{36}}$

$= \underline{6}$

(g) $\sqrt{2} \cdot \sqrt{32} = \sqrt{\underline{2} \cdot \underline{32}}$

$\sqrt{2} \cdot \sqrt{32} = \underline{\sqrt{64}}$

$= \underline{8}$

Lesson 16-8.

Negative integers are necessary in order to subtract integers. Likewise, negative irrational numbers are necessary for subtraction.

It is recommended that the teacher review subtraction of integers on the number line and the concept of rewriting subtraction problems as "addition of the opposite".

At the beginning of the lesson distribute only pages 16-8 and 16-8a. Distribute the other pages after students have completed Problem 3.

Class Discussion - Page 16-8.

4. (a) Point A

$\sqrt{5} - \sqrt{2} = \underline{\sqrt{5}} + \underline{{}^{-}\sqrt{2}}$

(b) Point B

$\sqrt{10} - \sqrt{3} = \underline{\sqrt{10}} + \underline{{}^{-}\sqrt{3}}$

(c) Point C

$\sqrt{7} - \sqrt{10} = \underline{\sqrt{7}} + \underline{{}^{-}\sqrt{10}}$

(d) Point D

$\sqrt{2} - \sqrt{6} = \underline{\sqrt{2}} + \underline{{}^{-}\sqrt{6}}$

5. ${}^{-}\sqrt{4} = {}^{-}2$

6. $({}^{-}2) \cdot ({}^{-}2) = 4$

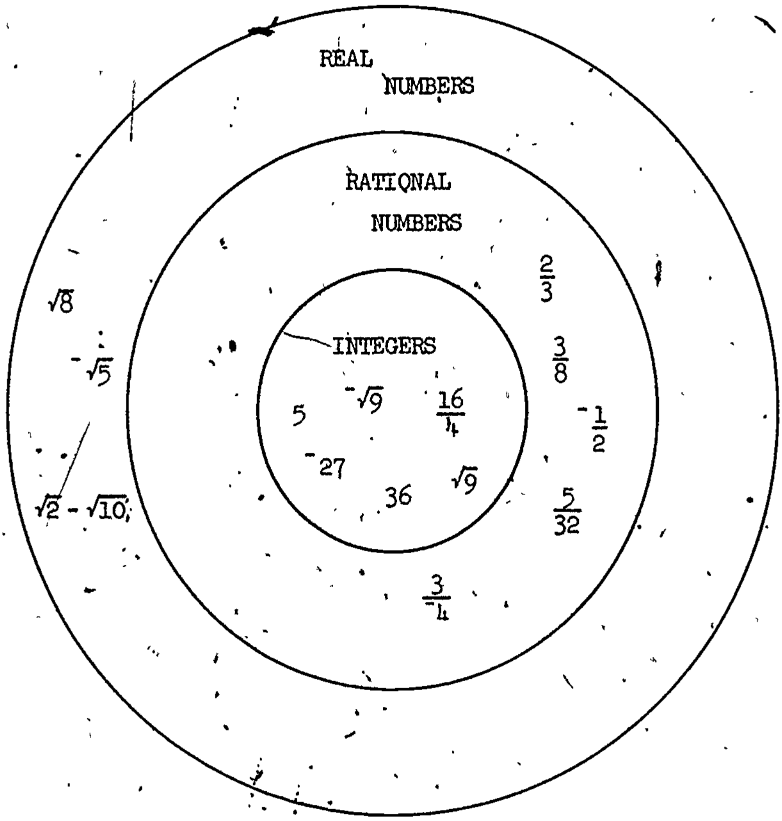
8. (a) Yes

(b) of 9 ? $\underline{3}$ and $\underline{{}^{-}3}$

9. of 16 ? $\underline{4}$ and $\underline{{}^{-}4}$

of 25 ? $\underline{5}$ and $\underline{{}^{-}5}$

Exercises - Page 16-8c.



Lesson 16-9.

The need for reciprocals of irrational square roots leads to the consideration of "rationalizing" denominators. It is through this technique that we locate reciprocals on the number line. It is this concept we wish to emphasize. Proficiency in rationalizing denominators is not expected. Treat this skill as exposure only.

Class Discussion - Page 16-9b.

... multiply it by one? No

... change the number? No

$\sqrt{2} \cdot \sqrt{2}$ is 2.

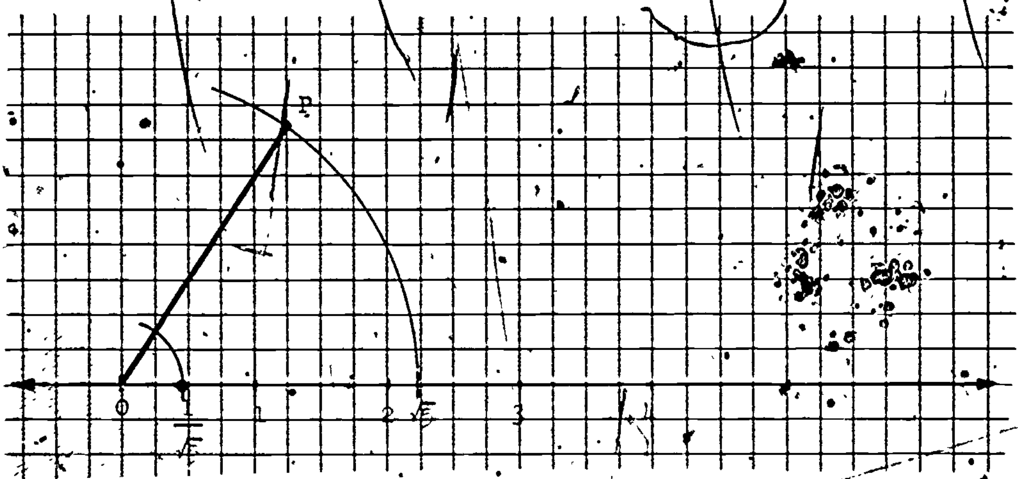
$\sqrt{3} \cdot \sqrt{3} = 3$

Exercises - Page 16-9e.

1. $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

2. 5

Construction for 3 through 7.



8. (a) $\frac{\sqrt{7}}{7}$

(b) $\frac{[illegible]}{2}$

(c) $\frac{2\sqrt{3}}{3}$

(d) $\frac{5\sqrt{6}}{6}$

(e) $\frac{\sqrt{30}}{5}$

(f) $\frac{4\sqrt{7}}{7}$

Lesson 16-10.

The teacher should have a supply of small circular objects available for students who do not have coins.

3.14, $3\frac{1}{7}$, and 3 are all just approximations of π. The choice of which approximation to use depends upon the circumstances of the application. The student should gain experience using each of these approximations.

Class Discussion - Page 16-10.

This point is a little to the right of ________.

Page 10a.

.... on their number line? Yes

On the number line on page 16-10 you used the diameter of the coin as your unit.

The circumference of the circle is a little more than 3 times longer than the diameter.

Exercises - Page 16-10b.

	D (diameter)	Rough estimate of C	Close approximation of C
1.	4 ft.	12 ft.	12.56 ft.
2.	5 in.	15 in.	15.70 in.
3.	8 cm	24 cm	25.12 cm
4.	7 yds.	21 yds.	21.98 yds.
5.	2.6 in.	7.8 in.	8.164 in.
6.	3.2 in.	9.6 in.	10.048 in.

Lesson 16-11.

Class Discussion - Page 16-11.

How many of these squares are there? 68

How many of these lightly shaded regions are there? 8

So about how many unit squares does this make? 8

... about $\frac{1}{2}$ of a unit square. How many of these regions are there? 4 So about how many unit squares does this make? 2

Number of dark squares is	68
Number of light squares is about	8
Number of unshaded squares is about	2
The area of the circle is about	78

Page 16-11b.

Area of this square = $4 \cdot (5)^2$

Area of this square = $4 \cdot 25$

Area of this square = 100 square units

... more or less than the area of the circle? More

Page 16-11d.

Here we need to interpret the area of the inscribed square in terms of the radius of the circle. The radius of the circle is $\frac{1}{2}$ the diagonal of this square so it is necessary to use the Pythagorean Theorem to find the area of the square.

Page 16-11d.

Area of this square = $2 \cdot \underline{(5)^2}$

Area of this square = $2 \cdot \underline{25}$

Area of this square = $\underline{50}$ square units

... more or less than the area of the circle? Less

$$3r^2 = 3 \cdot \underline{(5)^2}$$
$$= 3 \cdot \underline{25}$$
$$= \underline{75} \text{ square units}$$

... units you got by counting? Yes

... too big or too small? too small

... very much too small? No

Page 16-11g.

... a little more than 3 ? π

$$A_{(circle)} = \pi r^2$$
$$A_{(circle)} \approx 3.14 \cdot (5)^2$$
$$A_{(circle)} \approx 3.14 \cdot 25$$
$$A_{(circle)} \approx \underline{78.50}$$

Exercises - Page 16-11f.

1. $A = \pi r^2$

   $A \approx 3.14 \cdot (6)^2$

   $A \approx 3.14 \cdot 36$

   $A \approx 113.04$ square feet

2. $A = \pi r^2$

$A \approx \frac{22}{7} \cdot (14)^2$

$A \approx \frac{22}{7} \cdot 14 \cdot 14$

$A \approx 22 \cdot 2 \cdot 14$

$A \approx 616$ square cm

3. $A = \pi r^2$

$A \approx 3.14 \cdot (12)^2$

$A \approx 3.14 \cdot 144$

$A \approx 452.16$

4. (a) $r = \frac{1}{2} D$

$r = \underline{\ 21\ }$ inches

(b) $A = \pi r^2$

$A \approx \frac{22}{7} \cdot (21)^2$

$A \approx \frac{22}{7} \cdot 21 \cdot 21$

$A \approx 22 \cdot 3 \cdot 21$

$A \approx 1386$ square inches

Pre-Test Exercises - Page 16-P-1.

1. (b) $\sqrt{16}$

(c) $\sqrt{1}$

(e) $\sqrt{49}$

(h) $\sqrt{36}$

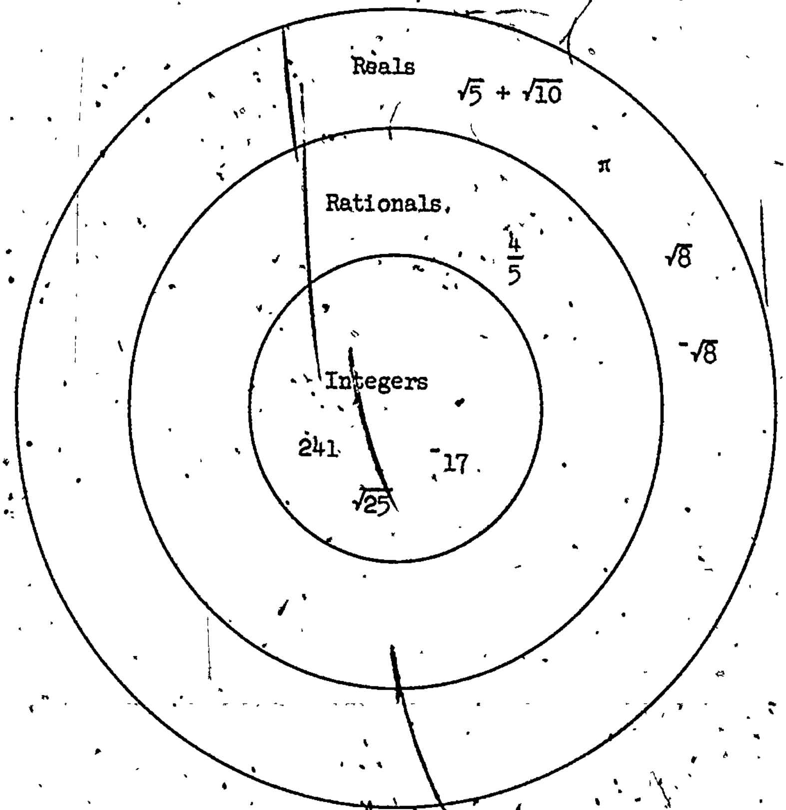
2. (a)
$a^2 + b^2 = c^2$
$\underline{5^2} + \underline{6^2} = c^2$
$\underline{25} + \underline{36} = c^2$
$\underline{61} = c^2$
$\underline{\sqrt{61}} = c$

(b)
$a^2 + b^2 = c^2$
$\underline{3^2} + \underline{7^2} = c^2$
$\underline{9} + \underline{49} = c^2$
$\underline{58} = c^2$
$\underline{\sqrt{58}} = c$

3. (e) none of these

4. (a) $\sqrt{55}$

5.



6. (a) $\frac{\sqrt{10}}{2}$

(b) $\frac{\sqrt{7}}{7}$

7. (a) $2\sqrt{5}$

(b) $5\sqrt{3}$

8. $A = \pi r^2$

$A \approx \frac{22}{7} \cdot (7)^2$

$A \approx \frac{22}{7} \cdot 7 \cdot 7$

$A \approx 22 \cdot 7$

$A \approx 154$ sq. in.

9. $C = \pi \cdot D$

$C \approx 3.14 \cdot 15$

$C \approx 47.10$ in.

10. 16 ft.

Test - Page 16-T-1.

1. (b) $\sqrt{64}$

(e) $\sqrt{49}$

2. $a^2 + b^2 = c^2$

$6^2 + 9^2 = c^2$

$36 + 81 = c^2$

$117 = c^2$

$\sqrt{117} = c$

3. (e) none of these

4. (c) $\sqrt{70}$

5. (a) $\sqrt{50} = 5\sqrt{2}$

(b) $\sqrt{8} = 2\sqrt{2}$

(c) $\sqrt{18} = 3\sqrt{2}$

(d) $\sqrt{12} = 2\sqrt{3}$

6. (a) $\frac{\sqrt{7}}{\sqrt{3}} = \underline{\frac{\sqrt{21}}{3}}$

(b) $\frac{\sqrt{2}}{\sqrt{5}} = \underline{\frac{\sqrt{10}}{5}}$

(c) $\frac{3}{\sqrt{7}} = \underline{\frac{3\sqrt{7}}{7}}$

(d) $\frac{5}{\sqrt{6}} = \underline{\frac{5\sqrt{6}}{6}}$

7. 13 ft.

8. 16 inches

9. $A = \pi r^2$

$A \approx 3.14 \cdot (9)^2$

$A \approx 3.14 \cdot 81$

$A \approx 254.34$ square feet

10. $C = \pi D$

$C \approx \frac{22}{7} \cdot 35$

$C \approx 22 \cdot 5$

$C \approx 110$ inches

Teacher's Commentary

Chapter 17

SOLVING EQUATIONS AND INEQUALITIES

This chapter shows two ways of solving equations and inequalities: (1) by writing equivalent statements, and (2) by graphing two functions.

The main idea, of course, is to help every student get the correct solution for each equation by the method that is easiest for him. Some students will probably use the "guess and try" method for fairly simple equations. It is hoped, however, that they will learn more efficient techniques when they encounter more complicated problems.

For some students it will be helpful to review the "box" method (Chapter 8). They should be allowed to use "boxes" as long as they need them.

In any event, do not insist that students write more than they have to in order to solve equations. If they can combine steps or do part or all of the work mentally, accept any method they use. Formal presentation of the steps is not the goal; getting the right answer is.

Notice that the idea of checking the solution for each equation or inequality is emphasized throughout the chapter. Here, again, it is not necessary for students to write the steps as formally as in the examples if they write enough to be sure the solution is correct.

Lesson 17-1.

Class Discussion - Page 17-1a.

$2 \cdot (3 + 4) = 7$ is not a true statement. You can make it true again by multiplying the number on the right side by $\underline{2}$.

$$2 \cdot (3 + 4) = \underline{2} \cdot 7$$

$(3 + 4) + 5 = 7$ is not a true statement. You can make it true again by adding 5 to the right side.

$$(3 + 4) + 5 = 7 + \underline{5}$$

1. (a) $3 \cdot (5 + 9) = \underline{3 \cdot} 14$
   (b) $(4 + 6) + 3 = 10 \underline{+ 3}$
   (c) $9 \cdot (26 + 8) = \underline{9 \cdot} 34$
   (d) $3 \cdot (15 + 45) + 8 = \underline{3 \cdot} 60 + \underline{8}$

To undo putting on your coat, you take off your coat. To undo picking up a book, you put the book down. To undo opening a door, you shut the door.

To undo adding 5, you add the opposite of 5.

To get back to 3 again, you add $\bar{}5$ to both sides.

On the right side you have 3.

2. You must add 16 to both sides.

$$x + 16 = 24 + 16$$
$$x = 40$$

3. You add $\bar{}59$ to both sides.

$$x + 59 + \bar{}59 = 65 + \bar{}59$$
$$x + 0 = 6$$
$$x = 6$$

To undo multiplying by 5, you multiply by the reciprocal of 5.

To get back to 4 again, you multiply both sides by $\frac{1}{5}$.

$$\frac{1}{5} \cdot 5 \cdot 4 = \frac{1}{5} \cdot 20$$
$$1 \cdot 4 = 4$$
$$4 = 4$$

4. You multiply both sides by $\frac{1}{10}$.

$$\frac{1}{10} \cdot 10x = \frac{1}{10} \cdot 90$$
$$1 \cdot x = 9$$
$$x = 9$$

5. You multiply both sides by 4.

$$4 \cdot \frac{1}{4} x = 4 \cdot 12$$
$$1 \cdot x = 48$$
$$x = 48$$

In $x + 9 = \bar{}3$, we add $\bar{}9$ to both sides.

$$x = \bar{}12$$

In $\frac{1}{5} x = 3$, we multiply both sides by 5 and learn that $x = 15$.

Exercises - Page 17-1e.

2. add 5. $x = 8$

3. multiply by $\frac{1}{5}$. $x = \frac{18}{5}$

4. multiply by 4. $x = 28$

5. multiply by 4. $x = 12$

6. add 19. $x = 6$

7. multiply by $\frac{1}{29}$. $x = 2$

8. multiply by $\frac{4}{3}$. $x = 16$

9. add $^{-}4$. $x = {}^{-}7$

10. multiply by $\frac{1}{5}$. $x = 10$

11. add 9. $x = 27$

12. multiply by $\frac{7}{5}$. $x = 21$

13. multiply by $\frac{8}{9}$. $x = 16$

14. add 50. $x = 50$

15. multiply by $\frac{1}{2}$. $x = 0$

Lesson 17-2.

Students often ask, "Why do I have to go through all these steps when I can just <u>see</u> the answer?" In an equation such as $2x + 3 = 11$, they probably see that $x = 4$ by mentally going through the same steps that we show on paper. This is the time to write an equation like $\frac{x}{6} - \frac{7}{2} = \frac{^{-}1}{3}$ and to ask whether the solution is obvious.

Class Discussion - Page 17-2.

$$2x + 3 + \underline{^{-}3} = 11 + \underline{^{-}3}$$

$$\underline{\tfrac{1}{2}} \cdot 2x = \underline{\tfrac{1}{2}} \cdot 8$$

In $7x + 4 = 46$, first both sides were multiplied by $\underline{7}$ and then $\underline{4}$ was added. To reverse the process, add the opposite of $\underline{4}$ to both sides and then multiply both sides by the reciprocal of $\underline{7}$.

In $\frac{5x}{7} + 5 = 15$, first both sides were multiplied by $\underline{\frac{5}{7}}$ and then $\underline{5}$ was added to both sides.

$$\frac{5}{7}x + 5 = 15$$

(Add $\underline{^{-}5}$).

$$\frac{5}{7}x = \underline{10}$$

(Multiply by $\underline{\frac{7}{5}}$).

$$x = \underline{14}$$

In $3x + {^{-}2} = 10$, you reverse the process by adding $\underline{2}$ to both sides and then multiplying both sides by $\underline{\frac{1}{3}}$.

$$3x + {^{-}2} = 10$$

(Add $\underline{2}$).

$$3x = \underline{12}$$

(Multiply by $\underline{\frac{1}{3}}$).

$$x = \underline{4}$$

In $5x + 2 = 22$, you will add $\underline{^{-}2}$ and then multiply by $\underline{\frac{1}{5}}$.

$$5x + 2 = 22$$
$$5x = \underline{20}$$
$$x = \underline{4}$$

To check:

$$(5 \cdot \underline{4}) + 2 = 22$$
$$\underline{20} + 2 = 22$$
$$\underline{22} = 22$$

Exercises - Page 17-2d. Check

1. (b) $2x + 1 = 41$ Add $^{-}1$. $(2 \cdot 20) + 1 = 41$

$2x = 40$ Multiply by $\frac{1}{2}$. $40 + 1 = 41$

$x = 20$ $41 = 41$

(c) $^{-}5x + 2 = 10$ Add $^{-}2$. $(^{-}5 \cdot \frac{^{-}8}{5}) + 2 = 10$

$5x = 8$ Multiply by $\frac{^{-}1}{5}$. $8 + 2 = 10$

$x = \frac{^{-}8}{5}$ $10 = 10$

(d) $\frac{x}{6} - \frac{7}{2} = \frac{^{-}1}{3}$ $(\frac{19}{6} - \frac{7}{2}) = \frac{^{-}1}{3}$

$\frac{1}{6} \cdot x + \frac{^{-}7}{2} = \frac{^{-}1}{3}$ $\frac{19}{6} + \frac{^{-}21}{6} = \frac{^{-}1}{3}$

$\frac{1}{6} \cdot x = \frac{19}{6}$ Add $\frac{7}{2}$. $\frac{^{-}2}{6} = \frac{^{-}1}{3}$

$x = 19$ Multiply by 6.

(e) $\frac{5x}{4} + 10 = 0$ Add $^{-}10$. $\frac{5 \cdot {^{-}8}}{4} + 10 = 0$

$\frac{5}{4}x = {^{-}10}$ Multiply by $\frac{4}{5}$. $\frac{^{-}40}{4} + 10 = 0$

$x = {^{-}8}$ $^{-}10 + 10 = 0$

Check

2. (b) $2x + 10 = 50$ $(2 \cdot 20) + 10 = 50$

$2x = 40$ $40 + 10 = 50$

$x = 20$ $50 = 50$

(c) $4x - 15 = 7$ $(4 \cdot \frac{11}{2}) + {^{-}15} = 7$

$4x + {^{-}15} = 7$ $\frac{44}{2} + {^{-}15} = 7$

$4x = 22$ $22 + {^{-}15} = 7$

$x = \frac{11}{2}$ $7 = 7$

(d) $2x + 5 = 5$ $(2 \cdot 0) + 5 = 5$

$2x = 0$ $0 + 5 = 5$

$x = 0$ $5 = 5$

(e) $\frac{2x}{5} + \frac{2}{5} = 2$

$\frac{2x}{5} = \frac{8}{5}$

$x = 4$

Check

$\frac{2 \cdot 4}{5} + \frac{2}{5} = 2$

$\frac{8}{5} + \frac{2}{5} = 2$

$\frac{10}{5} = 2$

$2 = 2$

(f) $^-3x + 10 = 1$

$^-3x = {}^-9$

$x = 3$

$(^-3 \cdot 3) + 10 = 1$

$^-9 + 10 = 1$

$1 = 1$

Lesson 17-3.

For students who question the idea that x cannot equal 0 in $\frac{3}{x} + 2 = \frac{7}{2}$, remind them that if $\frac{3}{0} = n$, then $n \cdot 0 = 3$, which is clearly impossible.

Class Discussion - Page 17-3.

To solve $3 \cdot \frac{1}{x} + 2 = \frac{7}{2}$, you first add $\underline{^-2}$ to both sides.

$$3 \cdot \frac{1}{x} + 2 + {}^-2 = \frac{7}{2} + \underline{^-2}$$

Next you multiply both sides by $\underline{\frac{1}{3}}$.

$$\frac{1}{3} \cdot 3 \cdot \frac{1}{x} = \underline{\frac{1}{3}} \cdot \frac{3}{2}$$

In $\frac{^-8}{x + 1} + 3 = 7$, you know that x cannot be $\underline{^-1}$ because $\underline{^-1} + 1 = 0$.

Finally add the opposite of [illegible] to both sides.

$$\frac{18}{x + 7} + \underline{3} = 5$$

$$18 \cdot \underline{\frac{1}{x + 7}} + \underline{3} = 5$$

Add the opposite of $\underline{3}$ to both sides.

$$18 \cdot \frac{1}{x + 7} = 2$$

Multiply both sides by the reciprocal of $\underline{18}$.

$$\frac{1}{x + 7} = \frac{2}{18} \quad \text{and} \quad \frac{2}{18} = \frac{1}{9}$$

$$\underline{x + 7} = 9$$

Add the opposite of $\underline{7}$ to both sides.

$$x = \underline{2}$$

To check:

$$\frac{18}{\underline{2} + 7} + {}^{-}3 = 5$$

$$\frac{18}{9} + {}^{-}3 = 5$$

$$2 + {}^{-}3 = 5$$

$$5 = 5$$

Exercises - Page 17-3c.

1. (a)

$$\frac{3}{x} - 2 = \frac{{}^{-}13}{8}, \text{ and } x \neq 0$$

$$\frac{3}{x} + {}^{-}2 = \frac{{}^{-}13}{8}$$

$$3 \cdot \frac{1}{x} + {}^{-}2 = \frac{{}^{-}13}{8} \quad \left(\frac{3}{x} = 3 \cdot \underline{\tfrac{1}{x}}\right)$$

$$3 \cdot \frac{1}{x} = \frac{3}{8} \quad \text{(Add } \underline{2} \text{ to both sides.)}$$

$$\frac{1}{x} = \frac{1}{8} \quad \text{(Multiply both sides by } \underline{\tfrac{1}{3}}.\text{)}$$

$$x = \underline{8}$$

Check

$$\frac{3}{8} - 2 = \frac{{}^{-}13}{8}$$

$$\frac{3}{8} + \frac{{}^{-}16}{8} = \frac{{}^{-}13}{8}$$

$$\frac{{}^{-}13}{8} = \frac{{}^{-}13}{8}$$

(b)

$$\frac{2}{x+4} + \frac{1}{4} = \frac{1}{2}, \text{ and } x \neq {}^{-}4$$

$$2 \cdot \frac{1}{x+4} + \frac{1}{4} = \frac{1}{2} \quad \left(\text{Rewrite } \frac{2}{x+4} \text{ as } 2 \cdot \underline{\tfrac{1}{x+4}}.\right)$$

$$2 \cdot \frac{1}{x+4} = \frac{1}{4} \quad \text{(Add } \underline{{}^{-}\tfrac{1}{4}} \text{ to both sides.)}$$

$$\frac{1}{x+4} = \frac{1}{8} \quad \text{(Multiply both sides by } \underline{\tfrac{1}{2}}.\text{)}$$

$$x + 4 = 8 \quad \text{(If those two numbers are equal, then } \underline{x + 4 = 8}.\text{)}$$

$$x = 4 \quad \text{(Add } \underline{{}^{-}4} \text{ to both sides.)}$$

$$\frac{2}{4+4} + \frac{1}{4} = \frac{1}{2}$$

$$\frac{2}{8} + \frac{2}{8} = \frac{1}{2}$$

$$\frac{4}{8} = \frac{1}{2}$$

2. (a)

$$\frac{7}{11}x = 1$$

$$x = \frac{11}{7}$$

Check

$$\frac{7}{11} \cdot \frac{11}{7} = 1$$

Check

(b) $^-5x = {}^-3$ | $^-5 \cdot \frac{3}{5} = {}^-3$
$x = \frac{3}{5}$ | $\frac{^-15}{5} = {}^-3$

(c) $x + 29 = 6$ | $^-23 + 29 = 6$
$x = {}^-23$ | $6 = 6$

(d) $4x + 8 = 10$ | $4 \cdot \frac{1}{2} + 8 = 10$
$4x = 2$ | $2 + 8 = 10$
$x = \frac{1}{2}$ | $10 = 10$

(e) $\frac{3}{5}x - 4 = {}^-1$ | $\frac{3}{5} \cdot 5 + {}^-4 = {}^-1$
$\frac{3}{5}x + {}^-4 = {}^-1$ | $\frac{15}{5} + {}^-4 = {}^-1$
$\frac{3}{5}x = 3$ | $3 + {}^-4 = {}^-1$
$x = 5$ | $^-1 = {}^-1$

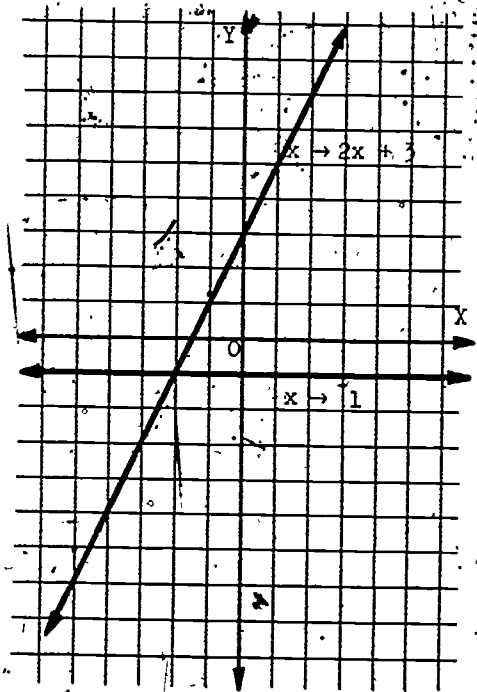
(f) $\frac{x}{5} + \frac{1}{4} = \frac{17}{4}$ | $\frac{20}{5} + \frac{1}{4} = \frac{17}{4}$
$\frac{1}{5} \cdot x + \frac{1}{4} = \frac{17}{4}$ | $4 + \frac{1}{4} = \frac{17}{4}$
$\frac{1}{5}x = 4$ | $\frac{16}{4} + \frac{1}{4} = \frac{17}{4}$
$x = 20$ | $\frac{17}{4} = \frac{17}{4}$

(g) $\frac{3}{x} + 6 = 7$, and $x \neq 0$ | $3 \cdot \frac{1}{3} + 6 = 7$
$3 \cdot \frac{1}{x} + 6 = 7$ | $1 + 6 = 7$
$3 \cdot \frac{1}{x} = 1$ | $7 = 7$
$\frac{1}{x} = \frac{1}{3}$
$x = 3$

Lesson 17-4.

Class Discussion - Page 17-4.

$f : x \to 2x + 3$	
Input	Output
1	5
0	3
$^{-}1$	1

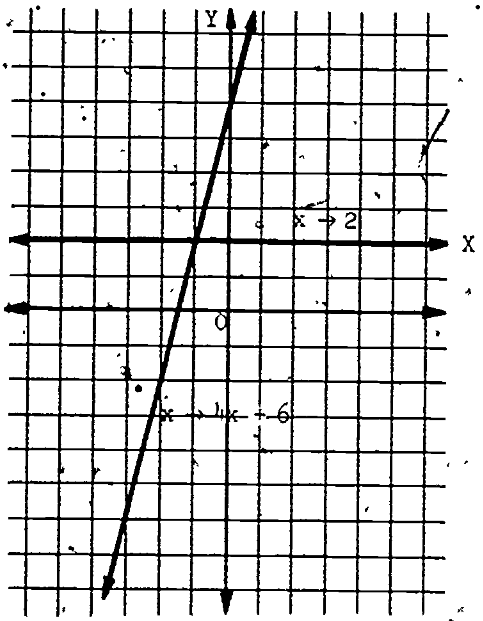


$g : x \to {}^{-}1$	
Input	Output
$\cancel{1}$	$^{-}1$
0	$^{-}1$
$^{-}1$	$^{-}1$

In $g : x \to {}^{-}1$, the output is always $\underline{{}^{-}1}$.

The graph of $f : x \to 2x + 3$ intersects the graph of $g : x \to {}^{-}1$ for an input of $\underline{{}^{-}2}$.

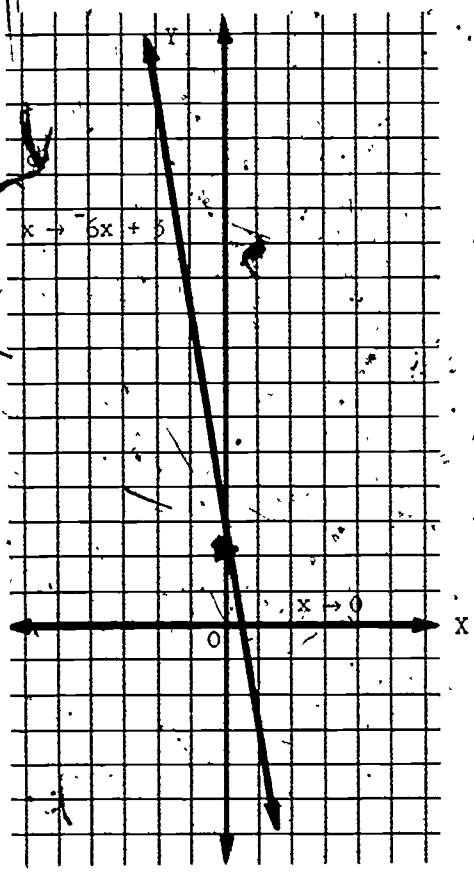
For the equation $4x + 6 = 2$, $f : x \to \underline{4x + 6}$ and $g : x \to \underline{2}$.



The input that gives the same output for both functions is $\underline{\ ^{-}1\ }$.

$$^{-}4x + 6 = 2$$
$$^{-}4x = ^{-}4$$
$$x = ^{-}1$$

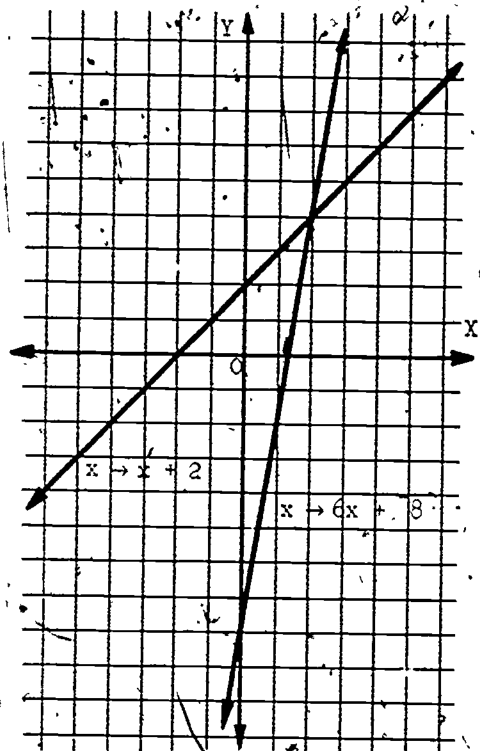
For the equation $^{-}6x + 3 = 0$, the functions are $f : x \rightarrow \underline{^{-}6x + 3}$ and $g : x \rightarrow \underline{0}$.



The solution of the equation $^{-}6x + 3 = 0$ is $x = \underline{\frac{1}{2}}$.

In order to show the output for an input of $^{-}5$, the Y-axis would have to be $\underline{33}$ units long.

For the equation $x + 2 = 6x + {}^{-}8$, the input that gives the same output for both functions is $\underline{2}$.



$\underline{2} + 2 = 6 \cdot \underline{2} + {}^{-}8$

$\underline{4} = \underline{12} + {}^{-}8$

$\underline{4} = \underline{4}$

Exercises - Page 17-4e.

1. $3x + {}^{-}1 = {}^{-}4$

(a) $f : x \rightarrow$ $\underline{3x + {}^{-}1}$

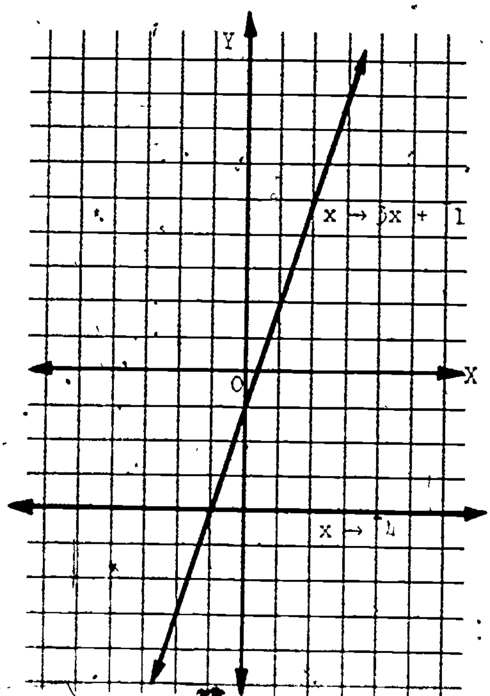
(b) $g : x \rightarrow$ $\underline{{}^{-}4}$

(c) ${}^{-}1$

(d) $(3 \cdot \underline{{}^{-}1}) + {}^{-}1 = {}^{-}4$

${}^{-}3 + {}^{-}1 = {}^{-}4$

${}^{-}4 = {}^{-}4$



2. $3x + 2 = 6x + {}^-4$

(a) $f : x \to \underline{3x + 2}$

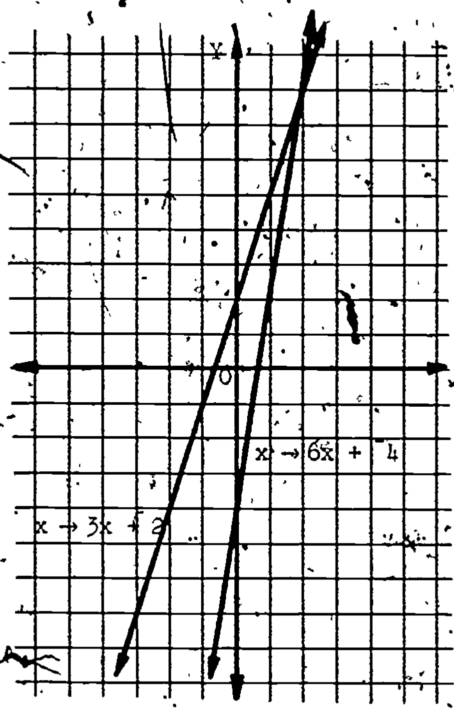
(b) $f : x \to \underline{6x + {}^-4}$

(c) 2

(d) $(3 \cdot \underline{2}) + 2 = (6 \cdot \underline{2}) + {}^-4$

$\underline{6} + 2 = \underline{12} + {}^-4$

$\underline{8} = \underline{8}$



3. $2x + {}^-1 = x + 4$

(a) $f : x \to \underline{2x + {}^-1}$

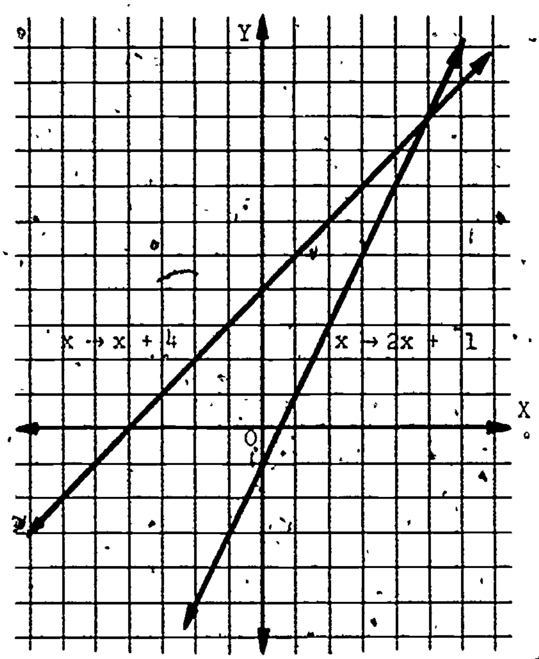
(b) $g : x \to \underline{x + 4}$

(c) 5

(d) $(2 \cdot \underline{5}) + {}^-1 = \underline{5} + 4$

$\underline{10} + {}^-1 = \underline{9}$

$\underline{9} = 9$



4. $2x + {}^-1 = 1 + {}^-2x$

(a) $f : x \rightarrow$ $\underline{2x + {}^-1}$

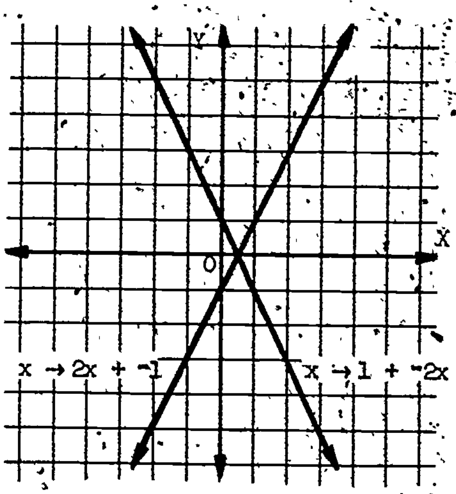
(b) $g : x \rightarrow$ $\underline{1 + {}^-2x}$

(c) $\frac{1}{2}$

(d) $(2 \cdot \frac{1}{2}) + {}^-1 = 1 + ({}^-2 \cdot \frac{1}{2})$

$1 + {}^-1 = 1 + {}^-1$

$0 = 0.$



Lesson 17-5.

In this section, students who have continued to use the "box" method will have to take one more step before they can use the boxes. It is possible that in their eagerness to get to the boxes they will incidentally learn to write equivalent equations. To solve $3x + 2 = 2x + 5$, the student will have to add ^-2x to both sides of the equation. It is not necessary to go into a formal discussion of the distributive property to convince students that $3x + {}^-2x = x$.

Class Discussion - Page 17-5.

You got rid of the 3 on the left side by adding $\underline{^-3}$ to both sides.

If you add the opposite of $\underline{6x}$, the right side is just $^-4$.

Value of x	Value of 6x	Opposite of 6x	^-6x
5	30	$^-30$	$^-30$
4	24	$^-24$	$^-24$
3	18	$^-18$	$^-18$
2	12	$^-12$	$^-12$
1	6	$^-6$	$^-6$
0	0	0	0
$^-1$	$^-6$	6	6
$^-2$	$^-12$	12	12
$^-3$	$^-18$	18	18

If you add $6x$ and ^-6x, the sum is $\underline{0}$.

The way to get rid of $6x$ is to add $\underline{^-6x}$.

Value of x	$3x + {}^-6x$	$(3 + {}^-6) \cdot x$
3	$9 + {}^-18 = {}^-9$	${}^-3 \cdot 3 = {}^-9$
2	$6 + {}^-12 = {}^-6$	${}^-3 \cdot 2 = {}^-6$
1	$3 + {}^-6 = {}^-3$	${}^-3 \cdot 1 = {}^-3$
0	$0 + 0 = 0$	${}^-3 \cdot 0 = 0$
$^-1$	${}^-3 + 6 = 3$	${}^-3 \cdot {}^-1 = 3$
$^-2$	${}^-6 + 12 = 6$	${}^-3 \cdot {}^-2 = 6$
$^-3$	${}^-9 + 18 = 9$	${}^-3 \cdot {}^-3 = 9$

Write ${}^-3x = {}^-6$, multiply both sides by $\frac{^-1}{3}$ and find the solution, $x = 2$.

Exercises - Page 17-5c

1. (a) ^-3x

   (b) $2x$

   (c) $\frac{^-1}{2}x$

   (d) $\frac{4}{3}x$

   (e) $10x$

   (f) ^-15x

2. (b) $2x + {}^-5x = (2 + {}^-5)x$
   $= {}^-3x$

   (c) $\frac{1}{2}x + \frac{1}{2}x = (\frac{1}{2} + \frac{1}{2})x$
   $= x$

   (d) $.5x + .7x = (.5 + .7)x$
   $= 1.2x$

   (e) $99x + {}^-97x = (99 + {}^-97)x$
   $= 2x$

(f) $\frac{3}{2}x + {}^{-}1x = (\frac{3}{2} + {}^{-}1)x$

$= \frac{1}{2}x$

(g) ${}^{-}3x + {}^{-}.5x = ({}^{-}3 + {}^{-}.5)x$

$= {}^{-}3.5x$

(h) $1.1x + {}^{-}.3x = (1.1 + {}^{-}.3)x$

$= .8x$

3. (b)

	$5x + 6 = 3x + {}^{-}6$	$(5 \cdot {}^{-}6) + 6 = (3 \cdot {}^{-}6) + {}^{-}6$
(Add ${}^{-}3x$.)	$5x + {}^{-}3x + 6 = {}^{-}6$	${}^{-}30 + 6 = {}^{-}18 + {}^{-}6$
(Add ${}^{-}6$.)	$5x + {}^{-}3x = {}^{-}12$	${}^{-}24 = {}^{-}24$
($5x + {}^{-}3x = \underline{2x}$)	$2x = {}^{-}12$	
(Multiply by $\frac{1}{2}$.)	$x = {}^{-}6$	

(c)

	${}^{-}x = 5x + {}^{-}3$	$\frac{{}^{-}1}{2} = (5 \cdot \frac{1}{2}) + {}^{-}3$
(Add ${}^{-}5x$.)	${}^{-}x + {}^{-}5x = {}^{-}3$	$\frac{{}^{-}1}{2} = \frac{5}{2} + {}^{-}3$
(${}^{-}x + {}^{-}5x = \underline{{}^{-}6x}$)	${}^{-}6x = {}^{-}3$	$\frac{{}^{-}1}{2} = \frac{{}^{-}1}{2}$
(Multiply by $\frac{{}^{-}1}{6}$.)	$x = \frac{1}{2}$	

(d)

	$3x + 8 = x + 40$	$(3 \cdot 16) + 8 = 16 + 40$
(Add ${}^{-}x$.)	$3x + {}^{-}x + 8 = 40$	$48 + 8 = 56$
(Add ${}^{-}8$.)	$3x + {}^{-}x = 32$	$56 = 56$
($3x + {}^{-}x = \underline{2x}$)	$2x = 32$	
(Multiply by $\frac{1}{2}$.)	$x = 16$	

(e)

	$5x + {}^{-}15 = x + 7$	$(5 \cdot \frac{11}{2}) + {}^{-}15 = \frac{11}{2} + 7$
(Add ${}^{-}x$.)	$5x + {}^{-}x + {}^{-}15 = 7$	$\frac{55}{2} + \frac{{}^{-}30}{2} = \frac{11}{2} + \frac{14}{2}$
(Add 15.)	$5x + {}^{-}x = 22$	$\frac{25}{2} = \frac{25}{2}$
($5x + {}^{-}x = \underline{4x}$)	$4x = 22$	
(Multiply by $\frac{1}{4}$.)	$x = \frac{22}{4} = \frac{11}{2}$	

4. (a)

$$4x = 3x + 105 \qquad (4 \cdot 105) = (3 \cdot 105) + 105$$
$$4x - 3x = 105 \qquad 420 = 315 + 105$$
$$x = 105 \qquad 420 = 420$$

(b)

$$4x + {}^-20 = 3x + 36 \qquad (4 \cdot 56) + {}^-20 = (3 \cdot 56) + 36$$
$$4x + {}^-3x + {}^-20 = 36 \qquad 224 + {}^-20 = 168 + 36$$
$$4x + {}^-3x = 56 \qquad 204 = 204$$
$$x = 56$$

(c)

$$2x + 1 = 3x + {}^-2 \qquad (2 \cdot 3) + 1 = (3 \cdot 3) + {}^-2$$
$$2x + {}^-3x + 1 = {}^-2 \qquad 6 + 1 = 9 + {}^-2$$
$$2x + {}^-3x = {}^-3 \qquad 7 = 7$$
$${}^-x = {}^-3$$
$$x = 3$$

(d)

$$3x + {}^-2 = 2x + 1 \qquad (3 \cdot 3) + {}^-2 = (2 \cdot 3) + 1$$
$$3x + {}^-2x + {}^-2 = 1 \qquad 9 + {}^-2 = 6 + 1$$
$$3x + {}^-2x = 3 \qquad 7 = 7$$
$$x = 3$$

(e)

$$.2x + .4 = 4 + {}^-x \qquad (.2 \cdot 3) + .4 = 4 + {}^-3$$
$$.2x + x + .4 = 4 \qquad .6 + .4 = 1$$
$$.2x + x = 3.6 \qquad 1 = 1$$
$$1.2x = 3.6$$
$$x = 3$$

Lesson 17-6.

Class Discussion - Page 17-6a:

1. (a) true
   (b) false
   (c) true
   (d) true
   (e) true
   (f) true
   (g) true
   (h) true

(i) true

(j) true

If $x < 1$, you can replace x with <u>0</u>.

If $x > 0$ you can replace x with <u>any whole number except zero</u>.

If $x + 1 < 5$, you can replace x with <u>0</u>, <u>1</u>, <u>2</u>, and <u>3</u>.

The solution set of $x + 1 < 5$ contains the whole numbers <u>0</u>, <u>1</u>, <u>2</u>, and <u>3</u>.

2. (a) 1

(b) 9

(c) 0, 1, 2, 3, 4, 5, 6, 7

(d) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

(e) 9

(f) 0, 1, 2, 3, 4, 5, 6, 7, 8

(g) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Exercises - Page 17-6c.

1. 0, 1, 2
2. 0
3. 10, 11, 12, ...
4. 8, 9, 10, ...
5. 22, 23, 24, ...
6. { }
7. 0, 1, 2, ..., 12
8. 0, 1, 2, ..., 11
9. 0, 1
10. 3, 4, 5, ...
11. 0, 1, 2
12. 0, 1, 2, 3, ..., 15
13. 16, 17, 18, ...
14. 0, 1, 2, 3, 4, 5
15. 0, 1, 2, 3, 4

16. 0 , 1 , 2 , 3 , 4

17. 0 , 1 , 2 , 3 , 4

18. 0 , 1 , 2 , 3

19. 2 , 3 , 4 , ...

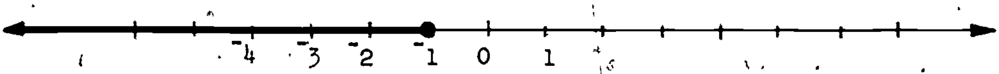
20. { }

Lesson 17-7.

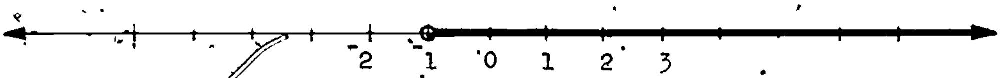
Class Discussion - Page 17-7.

The solution of the equation $2x + 3 = 11$ is $x = \underline{4}$.

For $2x + 3 < 11$, the solution set is $x \underline{< 4}$.

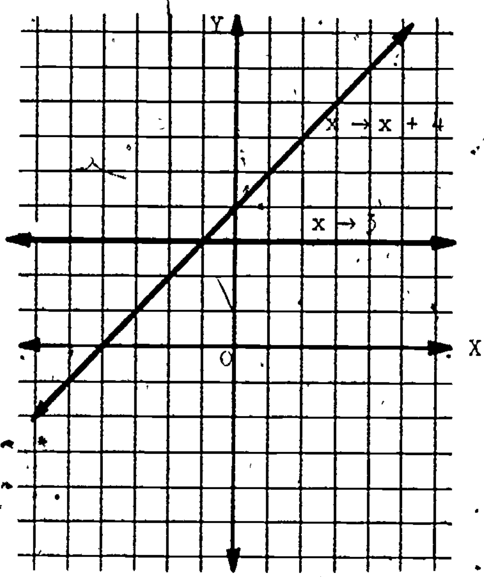


$x + 1 \leq 0$ and $x \leq {}^-1$



$x + 1 > 0$ and $x > {}^-1$

The functions for the equation $3 = x + 4$ are $f : x \rightarrow 3$ and $g : x \rightarrow x + 4$

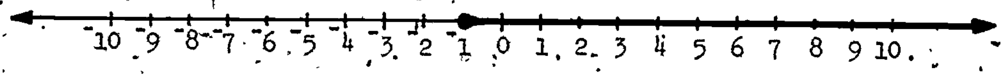


The input that gives the same output for both functions is $\underline{^-1}$.

With a number less than $^-1$, the statement is false; e.g., $3 < \underline{^-2} + 4$ is false.

With a number greater than $^-1$, the statement is true; e.g., $3 < \underline{0} + 4$.

The ray will go to the right.

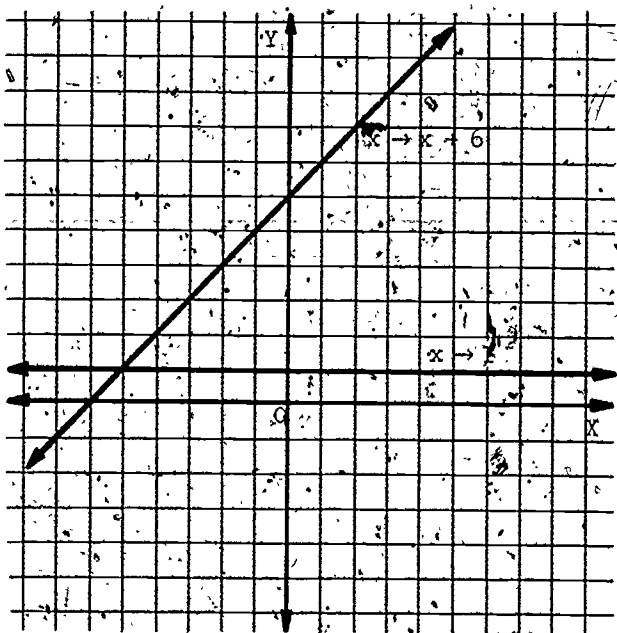


$x \geq {^-1}$

For the equation $x + 6 = 1$,

$f : x \to \underline{x + 6}$

$g : x \to \underline{\ 1\ }$

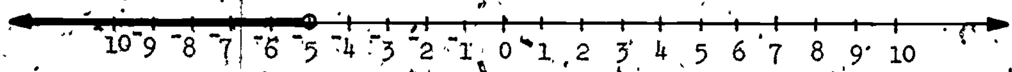


Solution of the equation: $x = {^-5}$

Numbers to the right are not in the solution set of $x + 6 < 1$.

Numbers to the left are in the solution set of $x + 6 < 1$.

The arrow points to the <u>left</u>.

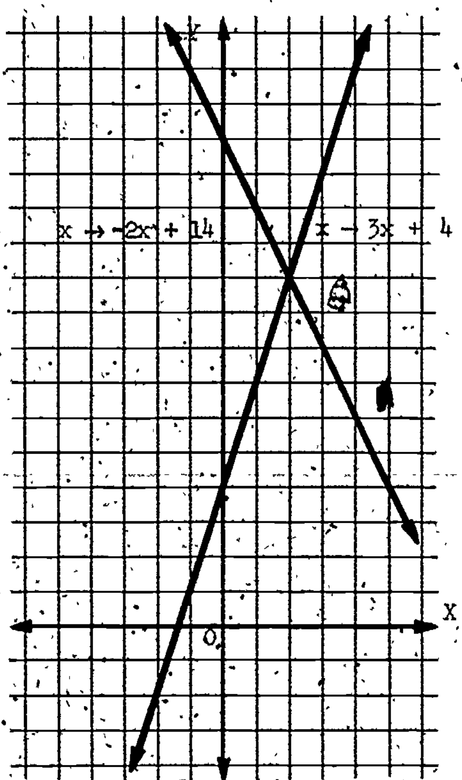


$$x < {}^-5$$

To solve $3x + 4 < {}^-2x + 14$:

$f: x \rightarrow 3x + 4$

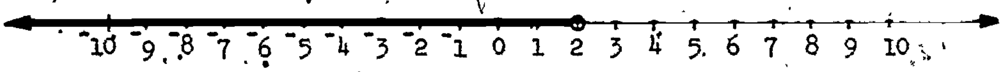
$g: x \rightarrow {}^-2x + 14$



The solution of the equation $3x + 4 = {}^-2x + 14$ is: $x = \underline{2}$.

A number greater than 2 makes the inequality <u>false</u>. A number less than 2 makes it <u>true</u>.

Solution set of $3x + 4 < {}^-2x + 14$:



$x < 2$

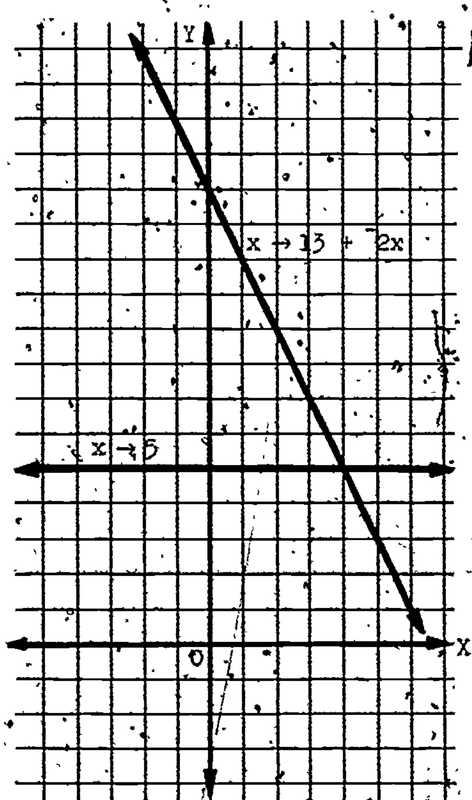
Exercises - Page 17-7f.

1. $13 + {}^-2x \geq 5$

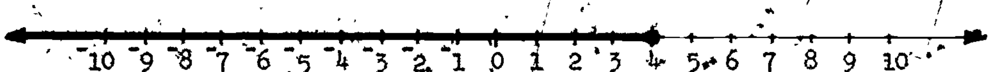
$\underset{\text{(Equation)}}{13 + {}^-2x = 5}$

$f : x \rightarrow \underline{13 + {}^-2x}$

$g : x \rightarrow \underline{\quad 5 \quad}$



Solution of equation: $x = \underline{4}$



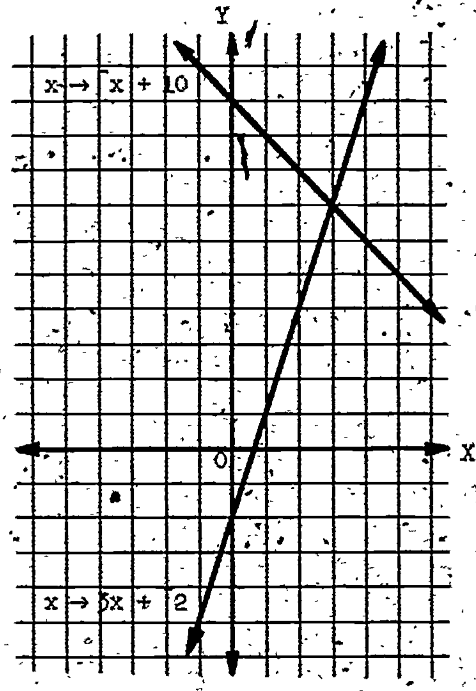
Solution set of $13 + {}^-2x \geq 5$: $\underline{x \leq 4}$

2. $3x + {}^{-}2 < {}^{-}x + 10$

$3x + {}^{-}2 = {}^{-}x + 10$
(Equation)

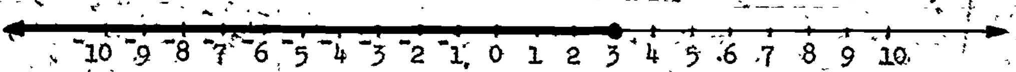
$f : x \to \underline{3x + {}^{-}2}$

$g : x \to \underline{{}^{-}x + 10}$



Solution of equation:

$x = \underline{\ 3\ }$



Solution set of $3x + {}^{-}2 < {}^{-}x + 10$:

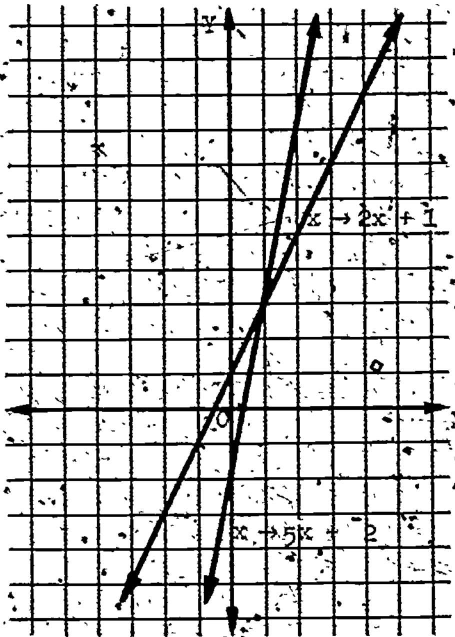
$\underline{x < 3}$

3. $5x + {}^{-}2 < 2x + 1$

$5x + {}^{-}2 < 2x + 1$
(Equation)

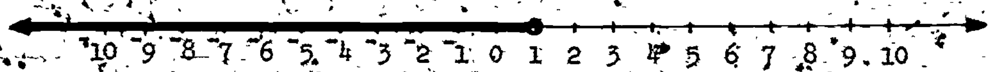
$f : x \to 5x + {}^{-}2$

$g : x \to 2x + 1$



Solution of equation:

$x = 1$



Solution set of $5x + {}^{-}2 < 2x + 1$:

$x < 1$

Lesson 17-8.

Class Discussion - Page 17-8.

1. $3 \underline{<} 5$
   $3 + 4 \underline{<} 5 + 4$

2. $^-9 \underline{<} 5$
   $^-9 + {}^-3 \underline{<} 5 + {}^-3$

3. $13 \underline{>} 7$
   $13 + 8 \underline{>} 7 + 8$

4. $4 \underline{>} {}^-1$
   $4 + 6 \underline{>} {}^-1 + 6$

5. $2 \underline{<} 7$
   $3 \cdot 2 \underline{<} 3 \cdot 7$

6. $^-3 \underline{<} 2$
   $4 \cdot {}^-3 \underline{<} 4 \cdot 2$

7. $8 \underline{>} {}^-1$
   $2 \cdot 8 \underline{>} 2 \cdot {}^-1$

8. $^-5 \underline{>} {}^-6$
   $1 \cdot {}^-5 \underline{>} 1 \cdot {}^-6$

9. $2 \underline{<} 7$
   $^-3 \cdot 2 \underline{>} {}^-3 \cdot 7$

10. $^-3 \underline{<} 2$
    $^-4 \cdot {}^-3 \underline{>} {}^-4 \cdot 2$

11. $8 \underline{>} {}^-1$
    $^-2 \cdot 8 \underline{<} {}^-2 \cdot 1$

12. $^-5 \underline{>} {}^-6$
    $^-1 \cdot {}^-5 \underline{<} {}^-1 \cdot {}^-6$

To solve $x + 3 < 5$ you add $\underline{^-3}$ to both sides to get $x + 0 < \underline{2}$, or simply $x < \underline{2}$.

To solve $x + {}^-5 > 8$ you add $\underline{5}$ to both sides to get $x + 0 > \underline{13}$ or $x > \underline{13}$.

13. (a) $x < {}^-6$

    (b) $x + {}^-8 < 21$
    $x < 29$

    (c) $x > 16$

    (d) $x + 2 < 14$
    $x < 12$

    (e) $x \leq 11$

    (f) $x \geq {}^-6$

In $4x < 12$ you multiply both sides by the reciprocal of $\underline{4}$. The solution is $x < \underline{3}$. In $\frac{4}{5}x > 20$, you multiply both sides by $\underline{\frac{5}{4}}$.

$x > \underline{25}$

14. (a) $x \geq \frac{1}{16}$

    (b) $x < 21$

    (c) $x \leq 9$

    (d) $x < {}^-5$

Exercises - Page 17-8d.

(Students may not use the same replacement for x to check their solutions.)

		Check
1.	$6x + 3 > 7 + 5x$	$(6 \cdot 5) + 3 > 7 + (5 \cdot 5)$
	$6x + {}^{-}5x + 3 > 7$	$30 + 3 > 7 + 25$
	$6x + {}^{-}5x > 4$	$33 > 32$
	$x > 4$	
2.	$5x + 11 < 3x + 3$	$(5 \cdot {}^{-}5) + 11 < (3 \cdot {}^{-}5) + 3$
	$5x + {}^{-}3x + 11 < 3$	${}^{-}25 + 11 < {}^{-}15 + 3$
	$5x + {}^{-}3x < {}^{-}8$	${}^{-}14 < {}^{-}12$
	$2x < {}^{-}8$	
	$x < {}^{-}4$	
3.	$100x + {}^{-}14 < 99x + {}^{-}10$	$(100 \cdot 3) + {}^{-}14 < (99 \cdot 3) + {}^{-}10$
	$100x + {}^{-}99x + {}^{-}14 < {}^{-}10$	$300 + {}^{-}14 < 297 + {}^{-}10$
	$100x + {}^{-}99x < 4$	$286 < 287$
	$x < 4$	
4.	$3x + {}^{-}2 < {}^{-}x + 10$	$(3 \cdot 2) + {}^{-}2 < {}^{-}2 + 10$
	$3x + x + {}^{-}2 < 10$	$6 + {}^{-}2 < 8$
	$3x + x < 12$	$4 < 8$
	$4x < 12$	
	$x < 3$	
5.	${}^{-}2x + 15 < x + {}^{-}12$	$({}^{-}2 \cdot 10) + 15 < 10 + {}^{-}12$
	${}^{-}2x + {}^{-}x + 15 < {}^{-}12$	${}^{-}20 + 15 < {}^{-}2$
	${}^{-}2x + {}^{-}x < {}^{-}27$	${}^{-}5 < {}^{-}2$
	${}^{-}3x < {}^{-}27$	
	$x > 9$	

Lesson 17-9.

This very light treatment of quadratic equations is designed only to: (1) introduce students to the idea of equations which are not linear, and (2) to provide one tool for finding at least an approximate solution to a quadratic equation.

Notice that the student is required to draw the graph of linear functions only.

Class Discussion - Page 17-9.

In $x^2 = 4$, $x = \underline{2}$, or $x = \underline{^{-}2}$.

If you want to get rid of $^{-}4$, you add $\underline{4}$.

$$x^2 + {}^{-}4 + \underline{4} = 5 + \underline{4}$$

so $$x^2 = \underline{9}$$

and $$x = \underline{3} \text{ or } x = \underline{^{-}3}.$$

To solve $2x^2 = 2$, we multiply both sides by $\underline{\frac{1}{2}}$.

$$\underline{\frac{1}{2}} \cdot 2x^2 = \underline{\frac{1}{2}} \cdot 2$$

so $$x^2 = \underline{1}$$

and $$x = \underline{1} \text{ or } x = \underline{^{-}1}$$

Exercises - Page 17-9a.

Notice that students may replace x^2 immediately in their checks for this exercise. Later they will have to use both values for x.

2. $\frac{x^2}{2} = 8$

$x^2 = \underline{16}$ $\qquad\qquad$ $\frac{16}{2} = 8$

$x = \underline{4}$ or $x = \underline{^{-}4}$ $\qquad\qquad$ $8 = 8$

3. $4x^2 + {}^{-}3 = 97$

$4x^2 = \underline{100}$ $\qquad\qquad$ $4 \cdot 25 + {}^{-}3 = 97$

$x^2 = \underline{25}$ $\qquad\qquad$ $100 + {}^{-}3 = 97$

$x = \underline{5}$ or $x = \underline{^{-}5}$ $\qquad\qquad$ $97 = 97$

4. $x^2 + {}^{-}17 = 32$

$x^2 = \underline{49}$ $\qquad\qquad$ $49 + {}^{-}17 = 32$

$x = \underline{7}$ or $x = \underline{^{-}7}$ $\qquad\qquad$ $32 = 32$

5. $3x^2 + 6 = 18$

$3x^2 = \underline{12}$

$x^2 = \underline{4}$

$x = \underline{2}$ or $x = \underline{^-2}$

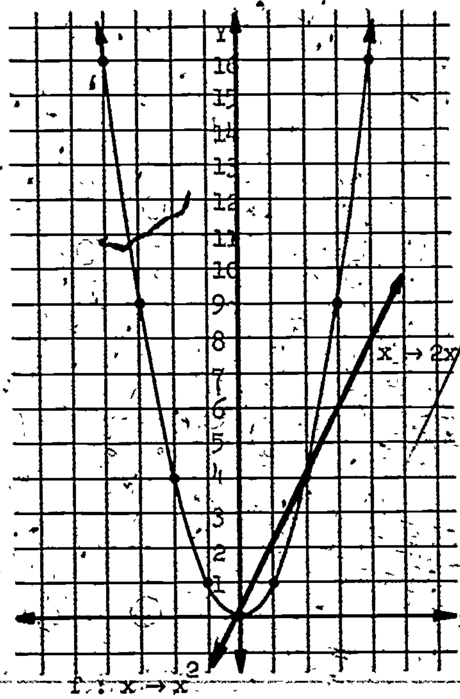
$3 \cdot 4 + 6 = 18$

$12 + 6 = 18$

$18 = 18$

Class Discussion - Page 17-9b.

Input	Output
x	x^2
$^-2$	4
$^-1$	1
0	0
1	1
2	4
3	9



There is no input that would give a negative output.

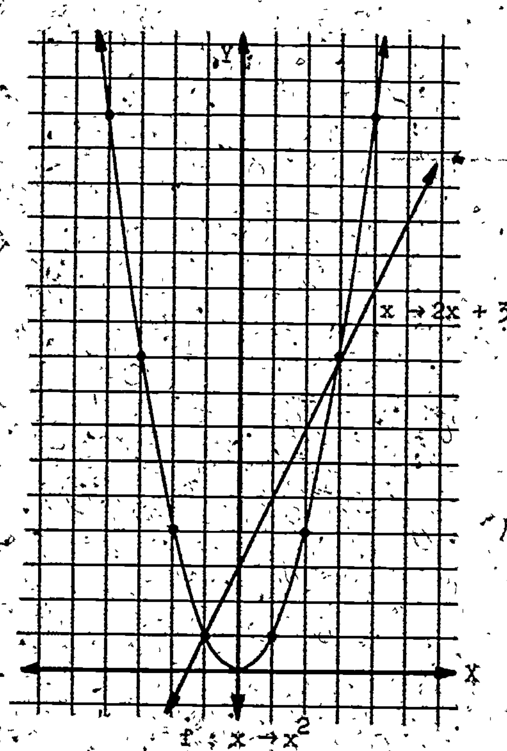
The product is always positive.

The graph of $g : x \to 2x$ intersects the graph of $f : x \to x^2$ at $\underline{2}$ points. Inputs that give the same outputs for both functions are $\underline{0}$ and $\underline{2}$. The solution set of the equation $x^2 = 2x$ is $x = 0$ or $x = 2$.

$$(\underline{0} \cdot \underline{0}) = (2 \cdot 0)$$

and $$(2 \cdot 2) = (2 \cdot 2)$$

For $x^2 = 2x + 3$, $f : x \to \underline{x^2}$ and $g : x \to \underline{2x + 3}$.



$x = {}^{-}1$ or $x = 3$

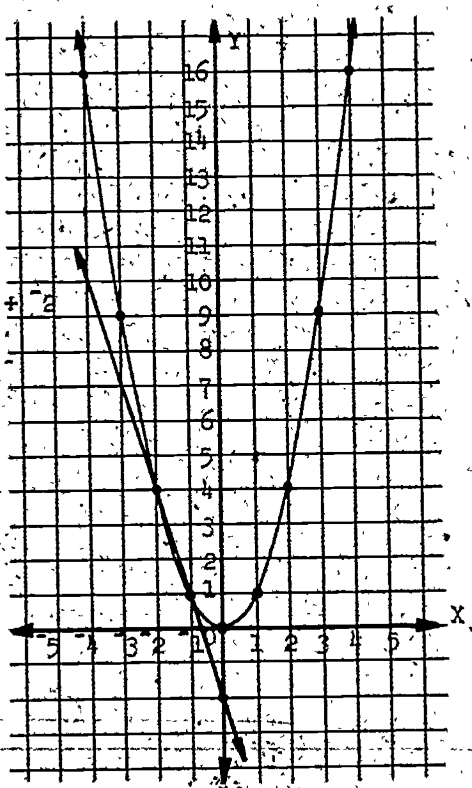
To get rid of ${}^{-}2x$ you add it's opposite and to get rid of ${}^{-}3$ you also add its opposite.

The solution for the equation $x^2 - 2x - 3 = 0$ is $x = {}^{-}1$ or $x = 3$.

For $x^2 + 3x + 2 = 0$, you add $\underline{^-3x}$ and $\underline{^-2}$.

$$x^2 = {}^-3x + {}^-2$$

$$g : x \rightarrow \underline{{}^-3x + {}^-2}$$



$x \rightarrow {}^-3x + {}^-2$

$f : x \rightarrow x^2$

The integers that give the same outputs are $\underline{^-1}$ and $\underline{^-2}$.

$(\underline{^-1} \cdot \underline{^-1}) + (3 \cdot \underline{^-1}) + 2 = 0$ and $(\underline{^-2} \cdot \underline{^-2}) + (3 \cdot \underline{^-2}) + 2 = 0$

$\underline{1} + \underline{^-3} + 2 = 0$ $\qquad$ $\underline{4} + \underline{^-6} + 2 = 0$

$\underline{0} = 0$ $\qquad$ $\underline{0} = 0$

Exercises - Page 17-9f.

Although only two blanks are provided for the check in each problem students should realize that they may use the space below these blanks also.

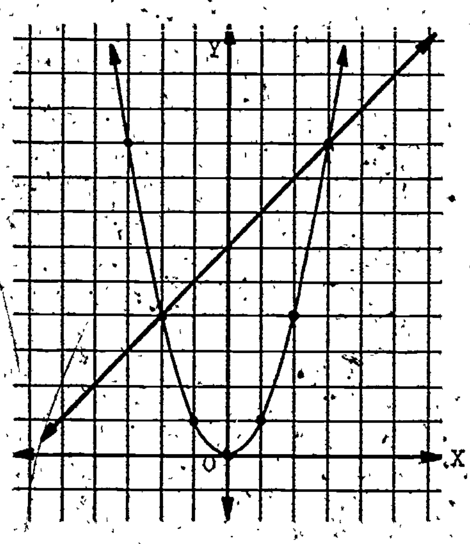
1. $x^2 - x - 6 = 0$

$x^2 + {}^-x + {}^-6 = 0$

$x^2 = \underline{x + 6}$ (Add $\underline{x}$ and $\underline{6}$.)

$f : x \to \underline{x^2}$

$g : x \to \underline{x + 6}$



$x = \underline{3}$ or $x = \underline{{}^-2}$

Check:

$(3 \cdot 3) + {}^-3 + {}^-6 = 0$

$9 + {}^-3 + {}^-6 = 0$

$0 = 0$

$({}^-2 \cdot {}^-2) + 2 + {}^-6 = 0$

$4 + 2 + {}^-6 = 0$

$0 = 0$

2. $x^2 - 3x - 4 = 0$

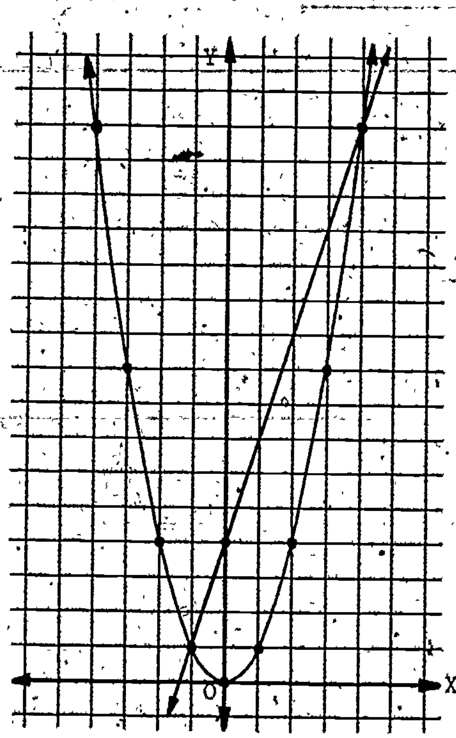
$x^2 + {}^-3x + {}^-4 = 0$

$x^2 = \underline{3x + 4}$

$f : x \to \underline{x^2}$

$g : x \to \underline{3x + 4}$

$x = \underline{{}^-1}$ or $x = \underline{4}$



Check:

$({}^-1 \cdot {}^-1) + 3 + {}^-4 = 0$

$1 + 3 + {}^-4 = 0$

$0 = 0$

$(4 \cdot 4) + {}^-12 + {}^-4 = 0$

$16 + {}^-12 + {}^-4 = 0$

$0 = 0$

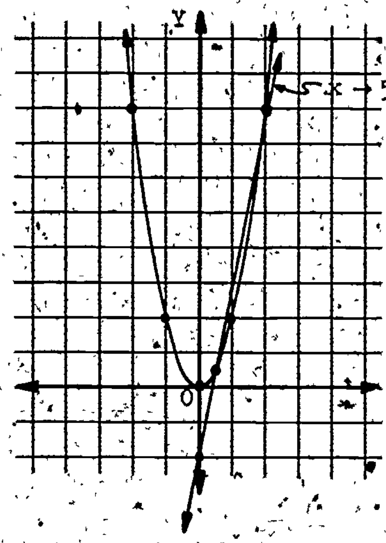
3. BRAINBOOSTER.

(a) $2x^2 + {}^-5x + 2 = 0$

$$2x^2 = 5x + {}^-2 \quad \text{(Add } 5x \text{ and } {}^-2\text{.)}$$

(b) $f : x \rightarrow 2x^2$

(c) $g : x \rightarrow 5x + {}^-2$



(d) The integer 2

$$2 \cdot (2 \cdot 2) + {}^-(5 \cdot 2) + 2 = 0$$
$$(2 \cdot 4) + {}^-10 + 2 = 0$$
$$8 + {}^-10 + 2 = 0$$
$$0 = 0$$

(e) A rational number between $\underline{0}$ and $\underline{1}$.

$$x = 2 \quad \text{or} \quad x = \frac{1}{2}$$

Pre-Test Exercises - Page 17-P-1

1. (a) Add $^{-}4$.
   (b) Add 3 .
   (c) Multiply by $\frac{1}{4}$.
   (d) Multiply by $^{-}2$.
   (e) Multiply by 3 .

2. (a) First add $^{-}9$, and then multiply by $\frac{1}{3}$.
   (b) First add 15 , and then multiply by 4 .
   (c) First add $^{-}49$, and then multiply by $\frac{16}{5}$.
   (d) First rewrite as $2x + {}^{-}3 = 10$, and then add 3 , and then multiply by $\frac{1}{2}$.

3. (a) 0
   (b) $\frac{1}{x}$
   (c) 4
   (d) 12

4. $f : x \to 2x + {}^{-}1$

   $g : x \to x + 3$

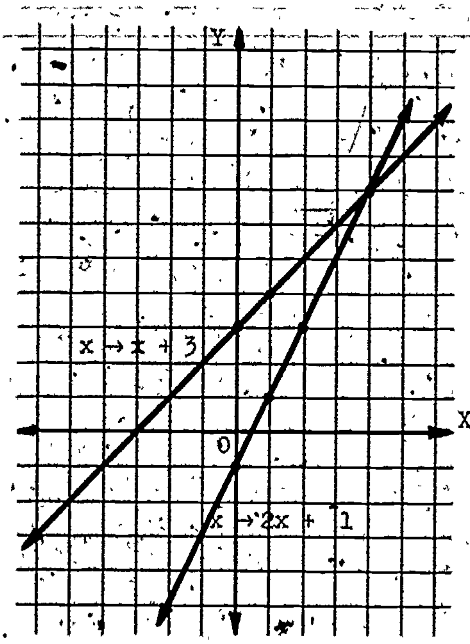
   $x = 4$

   Check

   $(2 \cdot 4) + {}^{-}1 = 4 + 3$

   $8 + {}^{-}1 = 4 + 3$

   $7 = 7$



Check:

5\. (a) $^-4x + 3 = 2x + 21$ $\quad$ $(4 \cdot {}^-3) + 3 = (2 \cdot {}^-3) + 21$

$^-4x + {}^-2x + 3 = 21$ $\quad$ $^-12 + 3 = {}^-6 + 21$

$^-4x + {}^-2x = 18$ $\quad$ $15 = 15$

$^-6x = 18$

$x = {}^-3$

(b) $7x - 4 = 2x + 21$ $\quad$ $(7 \cdot 5) - 4 = (2 \cdot 5) + 21$

$7x + {}^-4 = 2x + 21$ $\quad$ $35 - 4 = 10 + 21$

$7x + {}^-2x = 25$ $\quad$ $31 = 31$

$5x = 25$

$x = 5$

6\. (a) $x < 6$

(b) $x < {}^-\frac{1}{7}$

(c) $x \leq 35$

(d) $x > \frac{21}{4}$

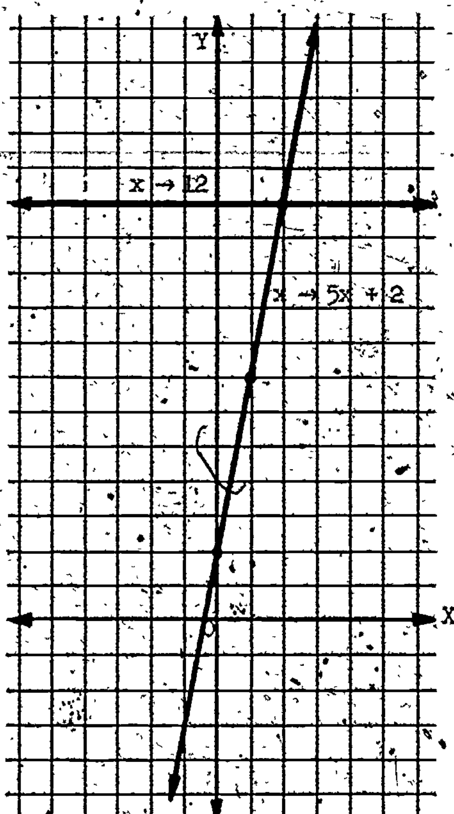
7\. (a) $5x + 2 < 12$

$5x + 2 = 12$ (Equation)

$f : x \rightarrow 5x + 2$

$g : x \rightarrow 12$

$x < 2$



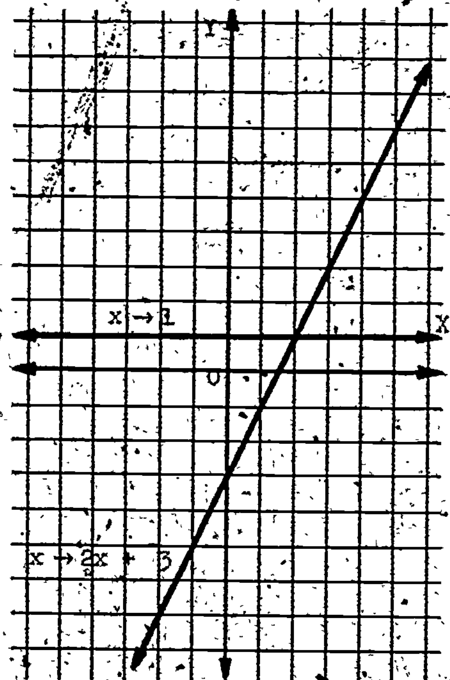
(b) $2x + {}^-3 \geq 1$

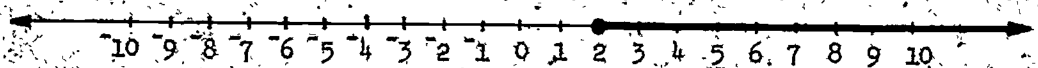
$2x + {}^-3 = 1$ (Equation)

$f : x \rightarrow 2x + {}^-3$

$g : x \rightarrow 1$

$x \geq 2$





(c) $3x + {}^-2 < 2x + 1$

$3x + {}^-2 < 2x + 1$ (Equation)

$f : x \rightarrow 3x + {}^-2$

$g : x \rightarrow 2x + 1$

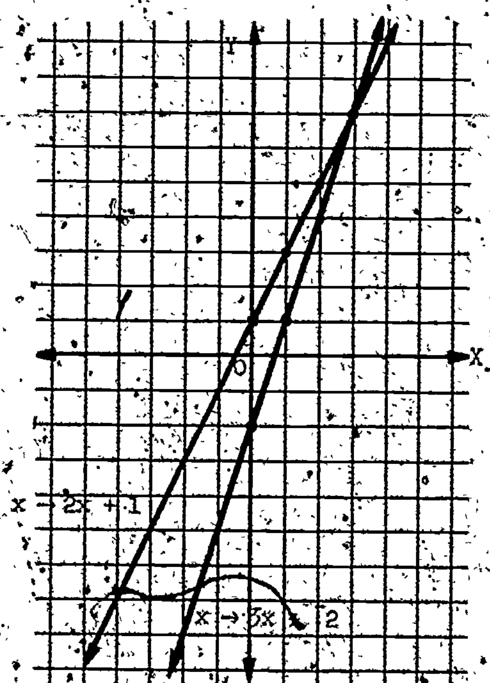
Solution of equation:

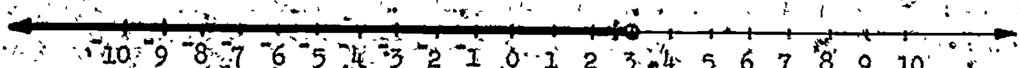
$x = 3$

Solution set of

$3x + {}^-2 < 2x + 1$

$x < 3$





8. (a) $4x + 3 < x + 6$

$4x + {}^-x + 3 < 6$

$4x + {}^-x < 3$

$3x < 3$

$x < 1$

Check

$4 \cdot 0 + 3 < 0 + 6$

$0 + 3 < 6$

$3 < 6$

(b) $3x + 8 < 5x - 2$

$3x + {}^-5x + 8 < {}^-2$

$3x + {}^-5x < {}^-10$

${}^-2x < {}^-10$

$x > 5$

$3 \cdot 6 + 8 < 5 \cdot 6 - 2$

$18 + 8 < 30 - 2$

$26 < 28$

9. $2x^2 - x - 3 = 0$

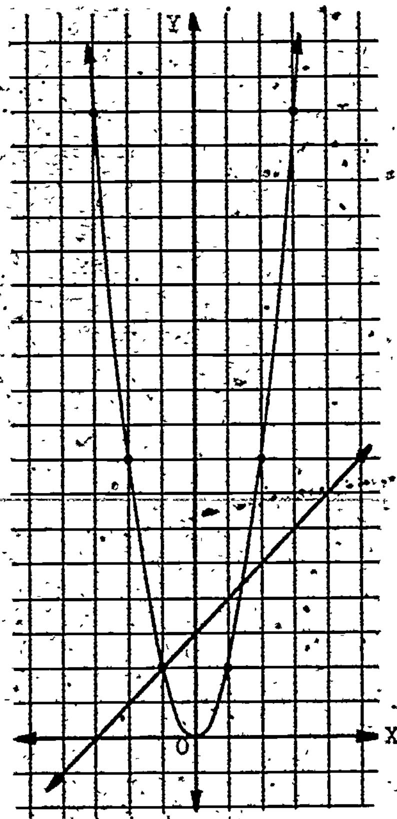
$2x^2 + {}^-x + {}^-3 = 0$

$2x^2 = x + 3$

$f : x \to 2x^2$

$g : x \to x + 3$

$x = {}^-1$ or $x = \frac{3}{2}$



$f : x \to 2x^2$

Check

$2 \cdot ({}^-1 \cdot {}^-1) + 1 + {}^-3 = 0$

$(2 \cdot 1) + 1 + {}^-3 = 0$

$2 + 1 + {}^-3 = 0$

$0 = 0$

and

$2 \cdot (\frac{3}{2} \cdot \frac{3}{2}) + \frac{^-3}{2} + {}^-3 = 0$

$2 \cdot \frac{9}{4} + \frac{^-3}{2} + {}^-3 = 0$

$\frac{9}{2} + \frac{^-3}{2} + {}^-3 = 0$

$0 = 0$

Test - Page 17-T-1

1. (a) Add 16

   (b) Multiply by $\frac{1}{3}$

   (c) Multiply by 4

   (d) Add 3

   (e) Multiply by 5

2. (a) $13x + 4 = 30$ | Check: $(13 \cdot 2) + 4 = 30$

$13x = 26$ | $26 + 4 = 30$

$x = 2$ | $30 = 30$

   (b) $\frac{5}{3}x - 6 = 19$ | $(\frac{5}{3} \cdot 15) + 6 = 19$

$\frac{5}{3}x + 6 = 19$ | $25 + 6 = 19$

$\frac{5}{3}x = 25$ | $19 = 19$

$x = 15$

   (c) $\frac{3}{2x} + 8 = \frac{83}{10}$ | $\frac{3}{2 \cdot 5} + 8 = \frac{83}{10}$

$\frac{3}{2} \cdot \frac{1}{x} + 8 = \frac{83}{10}$ | $\frac{3}{10} + 8 = \frac{83}{10}$

$\frac{3}{2} \cdot \frac{1}{x} = \frac{3}{10}$ | $\frac{3}{10} + \frac{80}{10} = \frac{83}{10}$

$\frac{1}{x} = \frac{1}{5}$ | $\frac{83}{10} = \frac{83}{10}$

$x = 5$

3. $4x + {}^-1 = 7$

$f : x \rightarrow 4x + {}^-1$

$g : x \rightarrow 7$

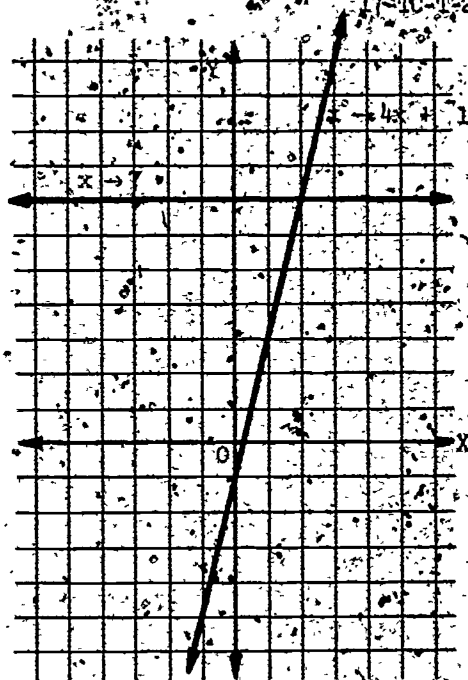
$x = 2$

<u>Check</u>

$(4 \cdot 2) + {}^-1 = 7$

$8 + {}^-1 = 7$

$7 = 7$



4. (b) ${}^-3x - 2 = 2x + {}^-7$

${}^-3x + {}^-2 = 2x + {}^-7$

$f : x \rightarrow {}^-3x + {}^-2$

$g : x \rightarrow 2x + {}^-7$

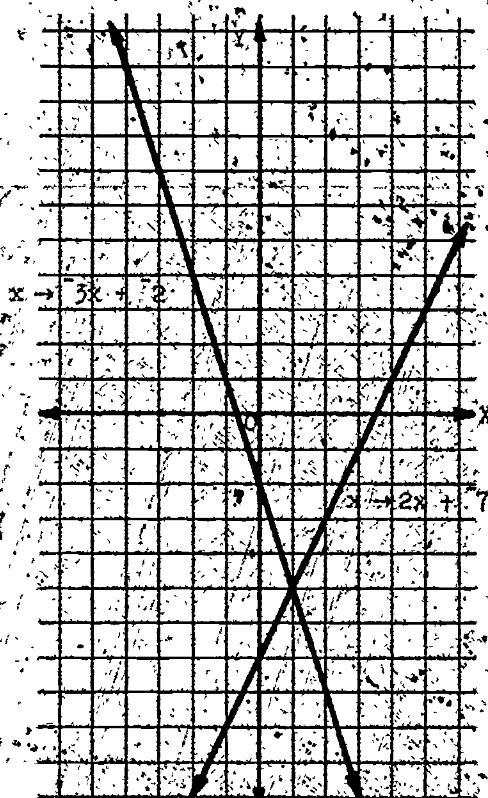
$x = 1$

<u>Check</u>

$({}^-3 \cdot 1) + {}^-2 = (2 \cdot 1) + {}^-7$

${}^-3 + {}^-2 = 2 + {}^-7$

${}^-5 = {}^-5$



4. $3x - 2 = {}^-x + 10$

$3x + {}^-2 = {}^-x + 10$

$3x + x + {}^-2 = 10$

$3x + x = 12$

$4x = 12$

$x = 3$

Check

$(3 \cdot 3) + {}^-2 = {}^-3 + 10$

$9 + {}^-2 = 7$

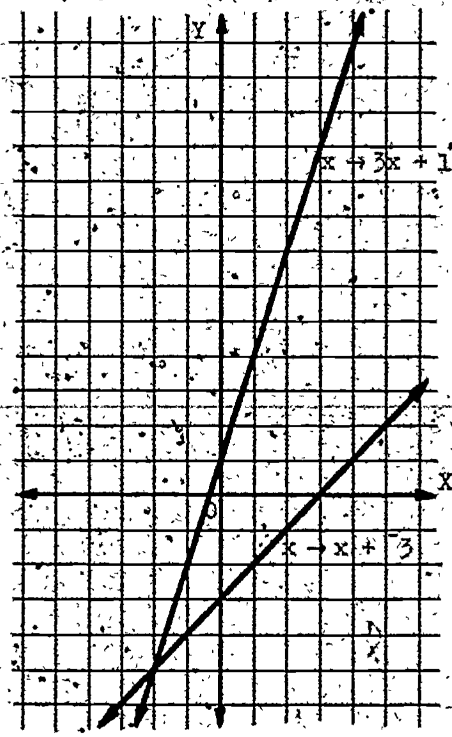
$7 = 7$

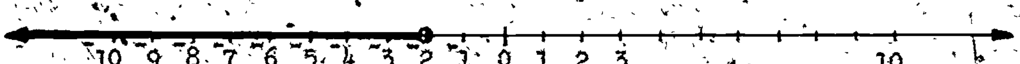
5. $x + {}^-3 > 3x + 1$

$x + {}^-3 = 3x + 1$

$f : x \rightarrow x + {}^-3$

$g : x \rightarrow 3x + 1$





6. ${}^-x + 2 \geq x + 3$

${}^-x + {}^-x + 2 \geq 3$

${}^-x + {}^-x \geq 1$

$x \leq {}^-\frac{1}{2}$

Check

${}^-({}^-1) + 2 \geq {}^-1 + 3$

$1 + 2 \geq {}^-1 + 3$

$3 \geq 2$

7. $x^2 - 2x - 3 = 0$

$x^2 + {}^-2x + {}^-3 = 0$

$x^2 = 2x + 3$

$f : x \to x^2$

$g : x \to 2x + 3$

$x = 3$ or $x = {}^-1$

<u>Check</u>

$(3 \cdot 3) + ({}^-2 \cdot 3) + {}^-3 = 0$

$9 + {}^-6 + {}^-3 = 0$

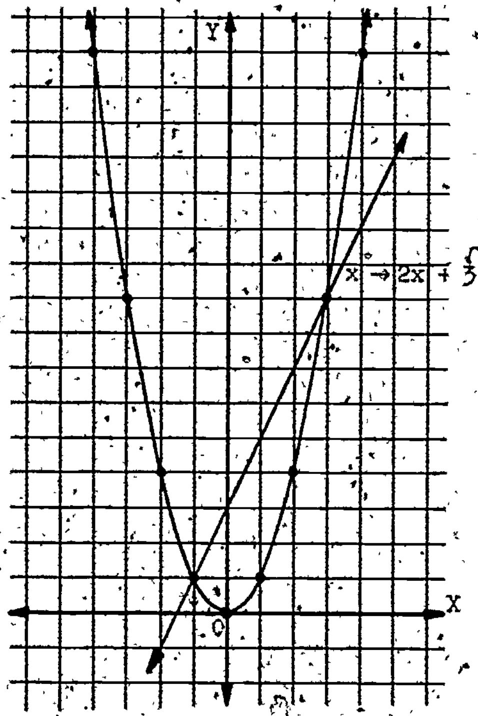
$0 = 0$

and

$({}^-1 \cdot {}^-1) + ({}^-2 \cdot {}^-1) + {}^-3 = 0$

$1 + 2 + {}^-3 = 0$

$0 = 0$



$f : x \to x^2$

211

Teacher's Commentary

Chapter 18

COORDINATE GEOMETRY

This final chapter of the series brings together many of the techniques and concepts developed in the previous seventeen chapters. Prior development of geometry, of functions and their graphs, and of algebraic techniques are united in a very intuitive manner and provide the teacher a final opportunity to reinforce gently many of the ideas developed under these topics.

Lesson 18-1.

There are three important ideas the student should obtain from this lesson.

(1) Given a function of the form $f : x \to mx + b$, the absolute value of m determines the slope ("steepness") of the line. The greater the absolute value of m, the closer the line approaches the vertical.

(2) The measure of the steepness of a line is called "the slope of the line". The slope may be found by dividing the "rise" by the "run". This quotient is the value of m in $f : x \to mx + b$.

(3) If, in determining the slope of a line you count up and then count over to the right the slope is positive. If you count up and then count over to the left the slope is negative.

Class Discussion - Page 18-1.

The graph of the function $h : x \to 4x$ is the steepest.

The graph of the function $k : x \to \frac{1}{2}x$ is the least steep.

Page 18-1b.

As the number that multiplies x gets larger the line gets steeper.

1. $g : x \to 3x$
2. $f : x \to 2x$
3. $p : x \to 14x$

4. $r : x \to \frac{3}{4}x$

5. $s : x \to \frac{3}{2}x$

Page 18-1c.

Yes. The origin (0,0) is on the line $f : x \to 2x$.

Yes. Counting up 2 units and then over 1 unit will put you back on the line.

You count over 1 unit to the right to come back to a point on the line.

The rise is 2. The run is 1.

1

If you start at any point on the line, count up 2 then 1 to the right, you will always come back to the line.

2

The rise is 4 and the run is 2.

$\frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$.

Yes. 2 is the slope.

Page 18-1d.

The slope of the line $g : x \to 3x$ is 3.

$\frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$.

Yes. 3 is the slope.

Page 18-1e.

Diagonal $\overline{OA}$.

rise = 6, run = 2

$\frac{\text{rise}}{\text{run}} = \frac{6}{2} = 3$. This is the slope.

Diagonal $\overline{FA}$. $\frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$

Diagonal $\overline{BO}$. $\frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$

Diagonal $\overline{EO}$. $\frac{\text{rise}}{\text{run}} = \frac{6}{2} = 3$

Diagonal $\overline{EB}$. $\frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$

Page 18-1f.

The rise over the run always gives the slope of a line.

For the graph of the function $k : x \rightarrow \frac{1}{2}x$, the <u>rise is</u> <u>1</u>.

The run is <u>2</u>. $\frac{\text{rise}}{\text{run}} = \frac{1}{2}$. The slope is $\underline{\frac{1}{2}}$.

Exercises - Page 18-1g.

1. (a) 4
   (b) 1
   (c) 4
2. (a) 1
   (b) 3
   (c) $\frac{1}{3}$

<u>Lesson 18-2</u>.

In this lesson we ask the student to look at the line and decide whether it leans to his left or to his right. Lines that lean to his left have <u>negative</u> slope. Lines that lean to his right have <u>positive</u> slope.

In the process of determining the slope of a line by dividing the rise by the run, students frequently forget that if you count up and then count over to the <u>left</u> the slope <u>is negative</u>. Knowing that if the line leans to his left the slope is negative provides a means for checking against this possible error.

Class Discussion - Page 18-2.

1. (a) right

   (b) positive

   (c) slope of ℓ_1 is $\frac{3}{4}$

   (d) $^{-}3$

Exercises - Page 18-2a-2c.

1. Slope = $\frac{5}{2}$
2. Slope = $\frac{^{-}4}{3}$
3. Slope = $\frac{^{-}3}{2}$
4. Slope = $^{-}5$
5. Slope = $\frac{1}{4}$
6. Slope = $^{-}1$
7. Slope = $\frac{^{-}1}{4}$
8. Slope = $\frac{3}{5}$

Lesson 18-3.

For the graph of any function of the form $f : x \to mx + b$, the constant b is the y-coordinate of the point where the line intersects the Y-axis. This is the y-intercept. Now, given the graph of some function the student can find the slope and the y-intercept and thus determine the function, because $f : x \to (\text{slope})x + (\text{y-intercept})$.

Also, given a function of the form $f : x \to mx + b$, the student now can easily draw its graph by first plotting the point whose y-coordinate is b and then use the slope, m, to plot a second point that lies on the line. Since two points determine a line he now can draw the graph without making a table of inputs and outputs for the function.

The rise for ℓ_2 is <u>3</u>.

The run for ℓ_2 is <u>1</u>.

The slope of ℓ_2 is <u>3</u>.

ℓ_2 crosses the Y-axis at a point with coordinates (0,2).

The y-coordinate of this point is <u>2</u>.

The y-intercept of ℓ_1 is <u>0</u>.

Class Discussion - Page 18-3b.

1. y-intercept of ℓ_1 is <u>-1</u>.
2. y-intercept of ℓ_2 is <u>0</u>.
3. y-intercept of ℓ_3 is <u>1</u>.
4. y-intercept of ℓ_4 is <u>2</u>.
5. y-intercept of ℓ_5 is <u>3</u>.
6. (a) <u>3</u>

   (b) <u>2</u>

   (c) Yes

   (d) No

   (e) Yes

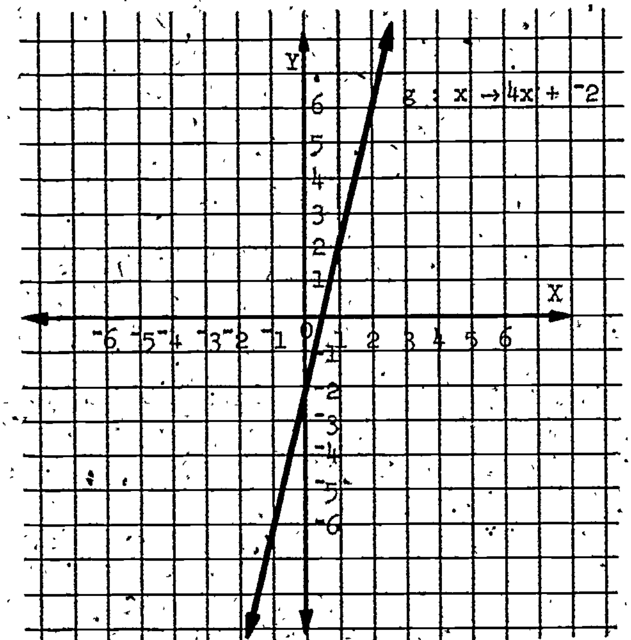
   (f) No

   (g) Yes

   (h) All points lie on line m.

   (i) Yes

7. (a)

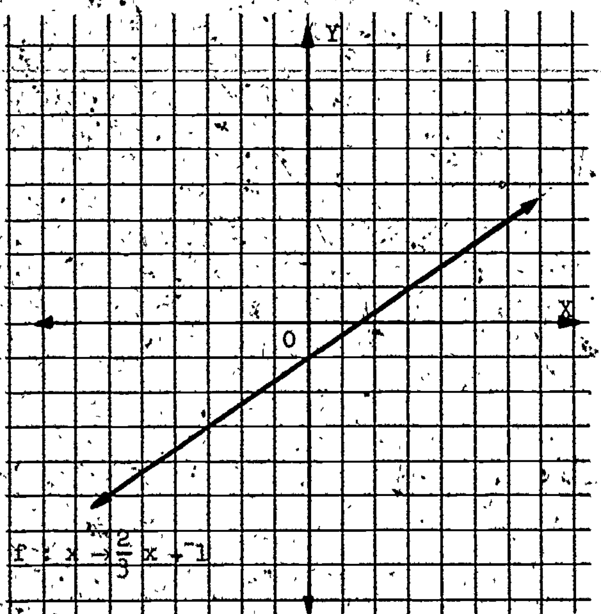


(b) The slope is 4 and the y-intercept is ⁻2.

8. $f : x \rightarrow 2x + 5$

9. $f : x \rightarrow \frac{3}{4}x + {}^{-}1$

10.



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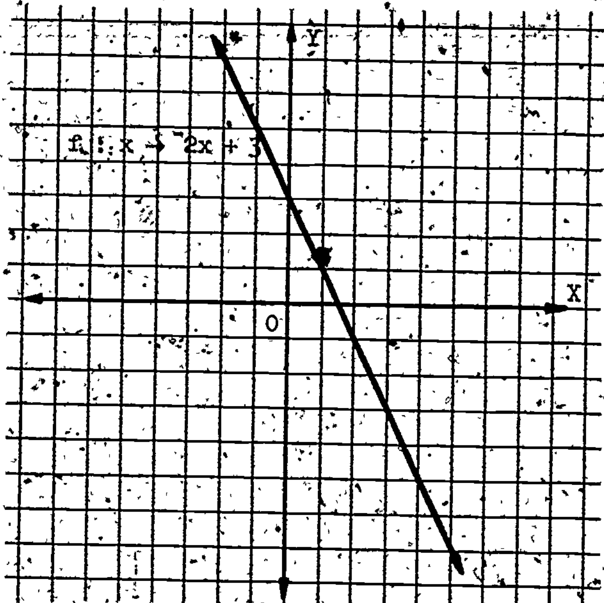

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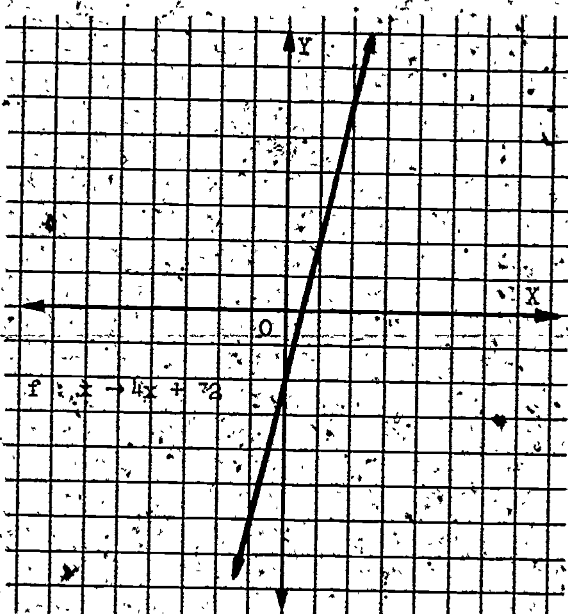


Exercises. - Page 18-3g.

1. (a) slope = 2

y-intercept is 3

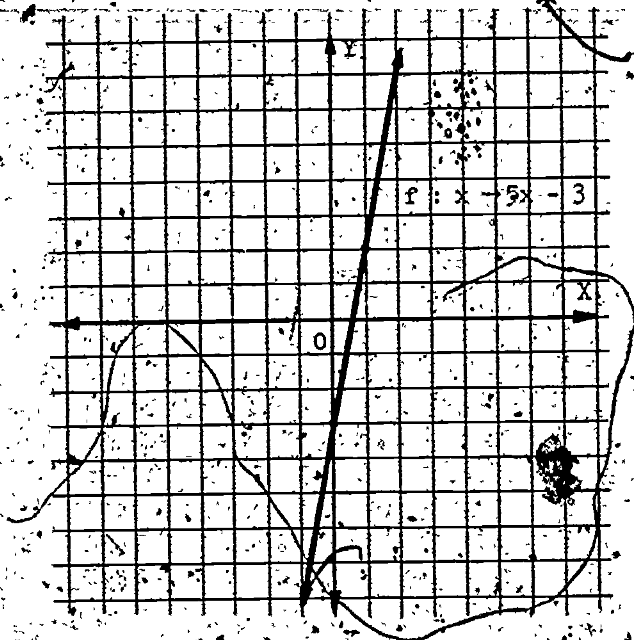
$f : x \rightarrow 2x + 3$

(b) slope = $\frac{1}{3}$

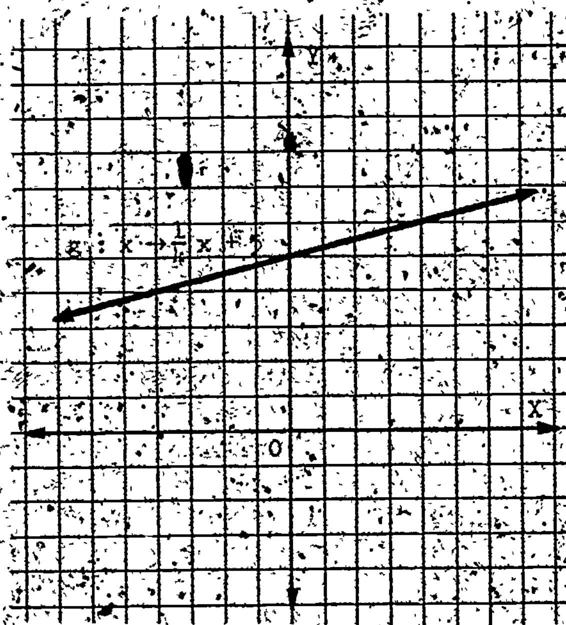
y-intercept is $^{-}2$

$f : x \rightarrow \frac{1}{3}x + {}^{-}2$

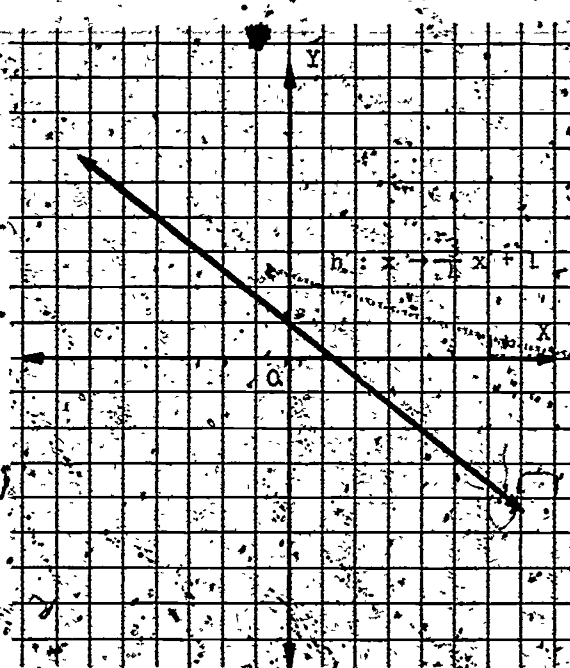
2. (a)



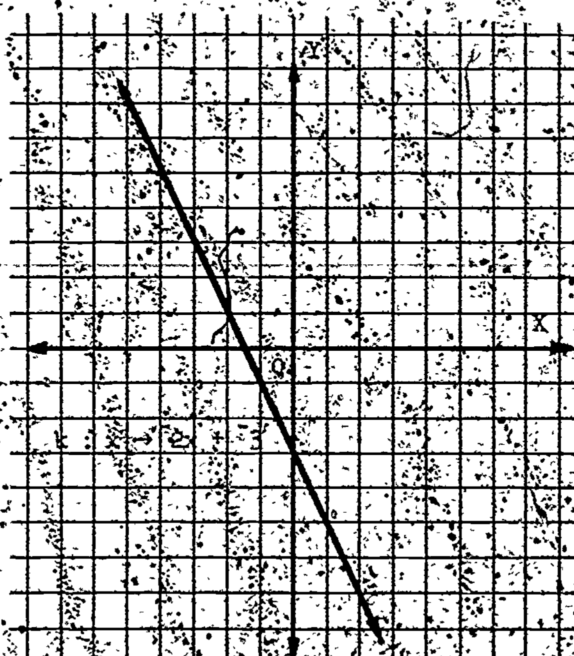
(b)



(c)



(d)



Lesson 18-4.

In this lesson, we draw the student's attention to the fact that if coordinates of a point in the coordinate plane are (x,y) then the y-coordinate is determined by the output of some function. That is, $y = mx + b$, where $mx + b$ is the output of the function $f : x \rightarrow mx + b$.

What we want the student to see is that any function of the form $f : x \rightarrow mx + b$ can be written as an equation of the form $y = mx + b$.

Class Discussion - Page 18-4.

1. x is the input.
2. $3x + 2$ is the output.
3. Y is the output axis.
4. y

Exercises - Page 18-4b.

1. (a) $\frac{5}{2}$
   (b) 1
   (c) $f : x \rightarrow \frac{5}{2}x + 1$
   (d) $y = \frac{5}{2}x + 1$

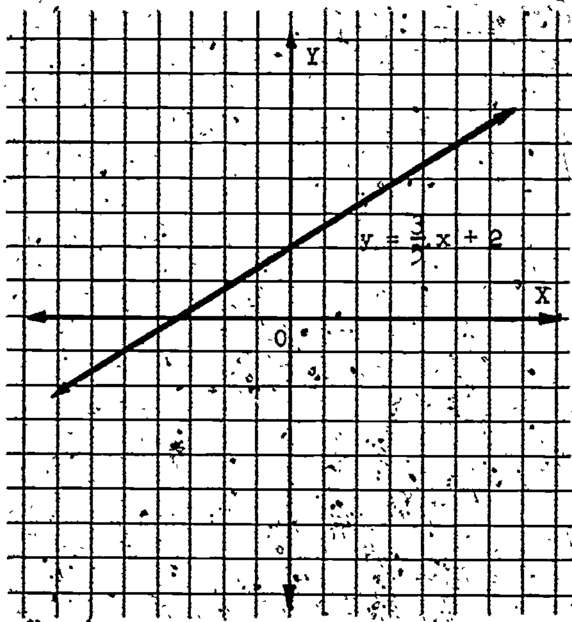
2. (a) $\frac{^-4}{3}$
   (b) $^-5$
   (c) $f : x \rightarrow \frac{^-4}{3}x + {}^-5$
   (d) $y = \frac{^-4}{3}x + {}^-5$

3. (a) $y = 2x + {}^-7$
   (b) $y = {}^-x + 4$
   (c) $y = \frac{7}{2}x + {}^-3$
   (d) $y = \frac{^-5}{6}x + 1$

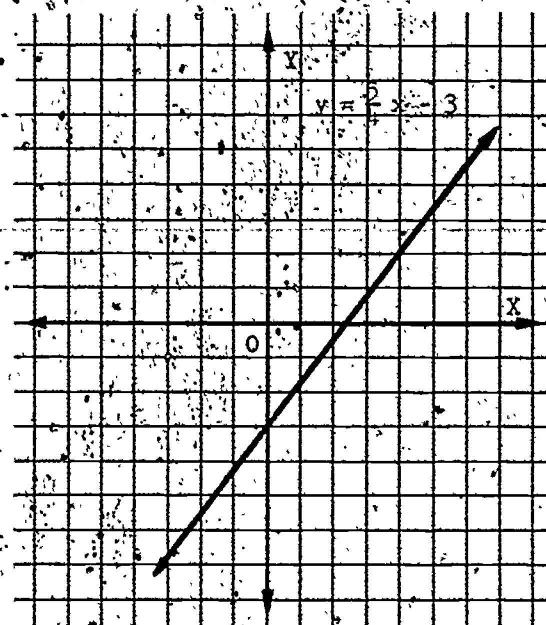
4. (a) $\frac{3}{5}$

(b) 2

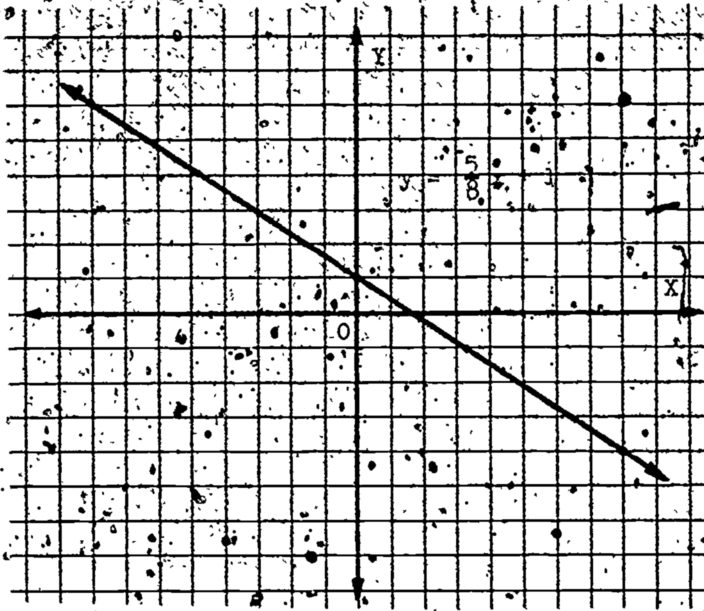
(c)



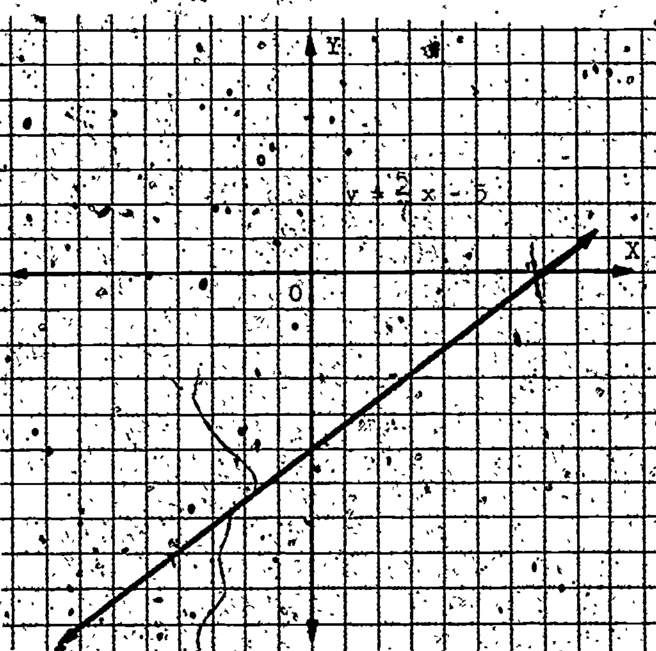
5. (a)



(b)



(c)



Lesson 18-5.

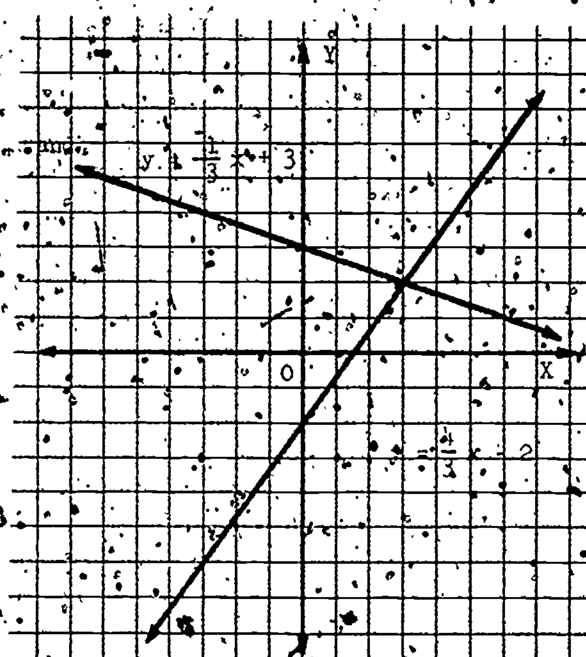
In Chapter 17, Equations, students learned that equations of the form $ax + b = cx + d$ could be solved by graphing the function for each side of the equation and finding the input (x-coordinate) of the point of intersection. This was the only number in the solution set for a linear equation with one variable, x.

In this section, however, you will have to make it clear that equations in the form $y = mx + b$ have two variables and thus have many solutions. In fact, the pair of coordinates of any point on the line is a solution. For the equation $y = 3x + 4$, the following pairs are all solutions: $(1,7)$, $(2,10)$, $(^-1,1)$, $(^-2,^-2)$, $(0,4)$, . . .

On the other hand, if we have the equations of two lines that intersect, then the pair of coordinates of the point of intersection is the only solution for that pair of equations. For, $y = 2x + {}^-3$ and $y = \frac{^-1}{2}x + 7$, the lines intersect at $(4,5)$, and the only pair that is a solution for both of the equations is $(4,5)$.

Class Discussion - Page 18-5.

1.



(a) (3,2)

(b) 3

(c) 2

(d) $y = \frac{4}{3}x + {}^-2$

$2 = \frac{4}{3} \cdot 3 + {}^-2$

$2 = 4 + {}^-2$

$2 = 2$

(e) <u>Yes</u>, (3,2) is a solution of $y = \frac{4}{3}x + {}^-2$.

(f) $y = \frac{{}^-1}{3}x + 3$

$2 = \frac{{}^-1}{3} \cdot 3 + 3$

$2 = {}^-1 + 3$

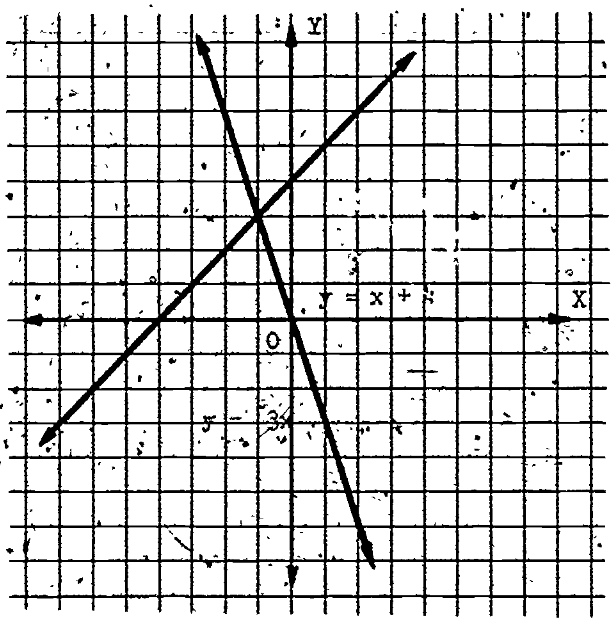
$2 = 2$

(g) <u>Yes</u>, (3,2) is a solution of $y = \frac{{}^-1}{3}x + 3$.

(h) <u>Yes</u>, (3,2) is the only solution of the <u>pair</u> of equations

$y = \frac{4}{3}x + {}^-2$ and $y = \frac{{}^-1}{3}x + 3$.

2.



(a) $(^-1,3)$

(b) $y = {}^-3x + 0$ and $y = x + 4$

$3 = {}^-3 \cdot {}^-1$ $\qquad$ $3 = {}^-1 + 4$

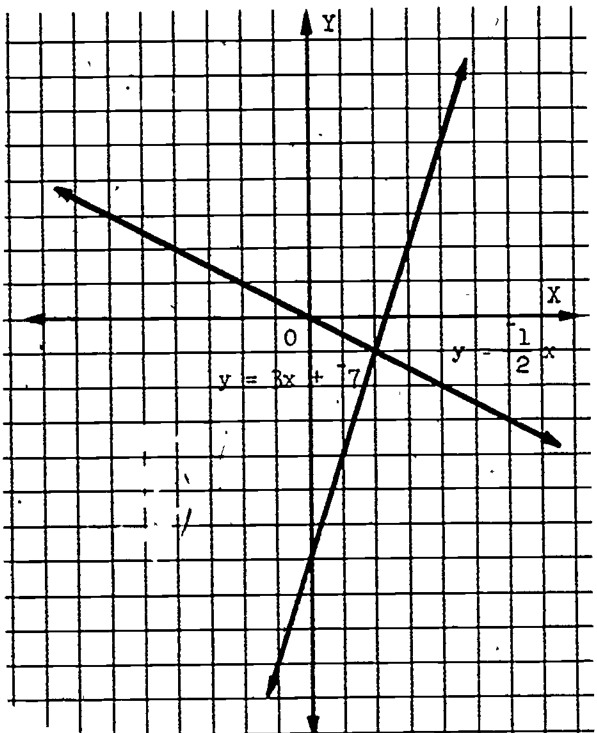
$\underline{3 = 3}$ $\qquad$ $\underline{3 = 3}$

(c) Yes, $(^-1,3)$ is the only solution of the pair of equations

$y = {}^-3x + 0$ and $y = x + 4$.

Exercises - Page 18-5c.

1. (a)



Solution: $\underline{(2,{}^-1)}$

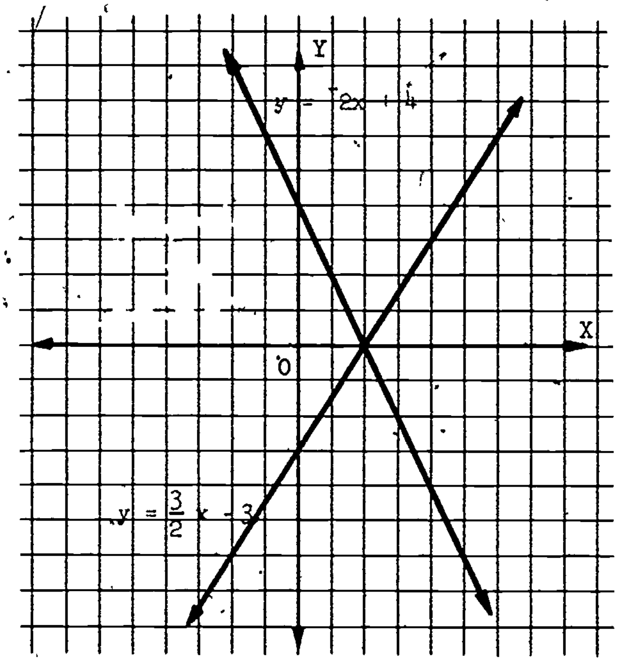
(b) $y = 3x + {}^-7$ and $y = \frac{^-1}{2}x + 0$

$^-1 = 3 \cdot 2 + {}^-7$ $\qquad$ $^-1 = \frac{^-1}{2} \cdot 2 + 0$

$^-1 = 6 + {}^-7$ $\qquad$ $^-1 = {}^-1$

$^-1 = {}^-1$

2. (a)



Solution: (2,0)

(b) $y = \frac{3}{2}x + {}^{-}3$ and $y = {}^{-}2x + 4$

$0 = \frac{3}{2} \cdot 2 + {}^{-}3$ $\qquad$ $0 = {}^{-}2 \cdot 2 + 4$

$0 = 3 + {}^{-}3$ $\qquad$ $0 = {}^{-}4 + 4$

$0 = 0$ $\qquad$ $0 = 0$

Lesson 18-6.

The success of this lesson is dependent upon the students actively participating in the class discussions. Be sure that the students trace the coordinate axes as carefully as possible.

The result of the class discussions should convince the student that if

$$m_1 \cdot m_2 = {}^{-}1$$

then the two lines are perpendicular, and if two lines are perpendicular then $m_1 \cdot m_2 = {}^{-}1$.

Class Discussion - Page 18-6a.

3. (a) Yes

   (b) $\frac{1}{4}$

   (c) $^{-}4$

   (d) $\frac{1}{4} \cdot {}^{-}4 = {}^{-}1$

4. (a) Yes

   (b) $\frac{1}{3}$

   (c) $^{-}3$

   (d) $\frac{1}{3} \cdot {}^{-}3 = {}^{-}1$

   (e) Yes

5. (a) Yes

   (b) $\frac{1}{2}$

   (c) $^{-}2$

   (d) $\frac{1}{2} \cdot {}^{-}2 = {}^{-}1$

   (e) Yes

6. (a) 1

   (b) $^{-}1$

   (c) $1 \cdot {}^{-}1 = {}^{-}1$

   (d) Yes

7. (a) 3

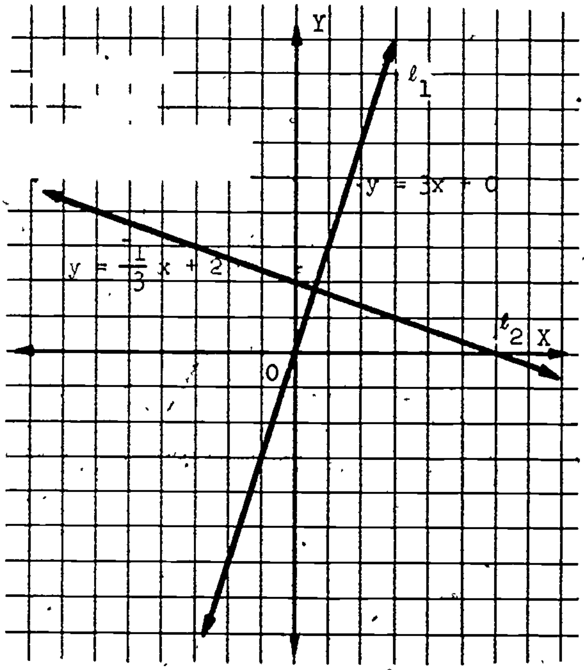
   (b) $\frac{1}{3}$

   (c) $\frac{^{-}1}{3}$

   (d) Yes

   (e) $y = \frac{^{-}1}{3} x + 2$

(f)



(g) Yes

8. (a) $\frac{2}{3}$

   (b) $\frac{3}{2}$

   (c) $\frac{^-3}{2}$

   (d) $y = \frac{^-3}{2}x + {}^-1$

9. (a) 1

   (b) 1

   (c) $^-1$

   (d) $y = {}^-x + 5$

Page 18-6e:

Any line of the form $y = c$ has a rise of 0.

The line $x = 3$ <u>is</u> a vertical line.

$x = 3$ is perpendicular to $y = 2$.

There is no number such that $x \cdot 0 = {}^-1$.

Exercises - Page 18-6f.

1. $y = 5x + 7 \quad \perp \quad$ (b) $y = \frac{-1}{5}x - 14$

$y = \frac{-1}{4}x + 2 \quad \perp \quad$ (d) $y = 4x - \frac{5}{6}$

$y = \frac{2}{3}x + 0 \quad \perp \quad$ (e) $y = \frac{-3}{2}x + \frac{2}{3}$

$y = \frac{-5}{8}x - \frac{4}{5} \quad \perp \quad$ (a) $y = \frac{8}{5}x - \frac{1}{7}$

$y = \frac{3}{7}x - \frac{1}{2} \quad \perp \quad$ (c) $y = \frac{-7}{3}x - \frac{1}{2}$

2. (a) The slope is 5 .

(b) $\frac{1}{5}$

(c) $\frac{-1}{5}$

(d) $y = \frac{-1}{5}x + 10$

3. $y = \frac{-3}{4}x + {}^{-}7$

4. $y = \frac{2}{3}x + \frac{7}{8}$

5. $y = \frac{1}{5}x + 14$

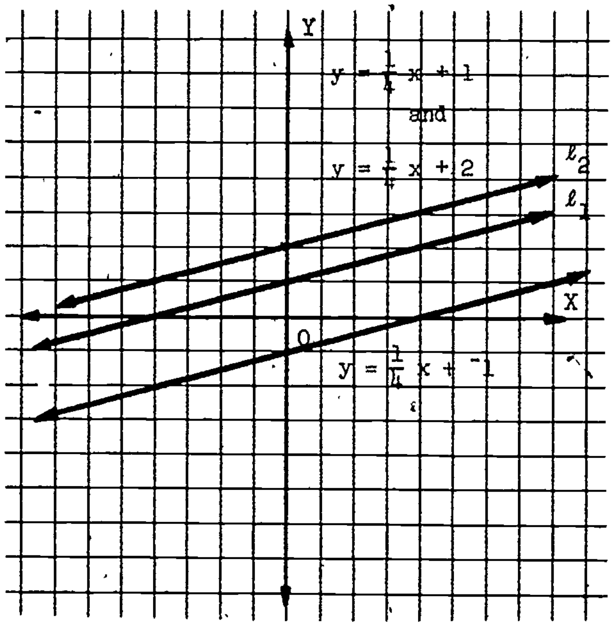
Lesson 18-7.

Class Discussion - Page 18-7.

1. (a) $y = \frac{1}{4}x + 1$

(b) $y = \frac{1}{4}x + 2$

(c)



(d) Yes

(e) Yes

(f) $\frac{1}{4}$

(g) $\frac{1}{4}$

(h) Yes

Exercises - Page 18-7a.

1.	$y = 3x - 2$	$\parallel$	(c) $y = 3x + 0$
	$y = \frac{3}{4}x + 5$	$\parallel$	(a) $y = \frac{3}{4}x - 12$
	$y = 17x + 6$	$\parallel$	(e) $y = 17x - 20$
	$y = \frac{1}{8}x - 7$	$\parallel$	(b) $y = \frac{1}{8}x + \frac{1}{2}$
	$y = {}^{-}5x + \frac{1}{6}$	$\parallel$	(d) $y = {}^{-}5x - \frac{3}{7}$

2. $y = \frac{2}{5}x + 8$

3. $y = 7x + {}^{-}5$

4. $y = \frac{5}{2}x + \frac{5}{8}$

Lesson 18-8.

The absolute value of a number is its distance from the origin on the number line, regardless of direction. This is one of the ideas we would like the student to understand. When the student sees $|{}^-3|$ we would like him to think "How far from zero is ${}^-3$?". Since distance cannot be negative, the obvious answer is "3" . With this understanding the student should see that if a and b are coordinates of two points on the number line, then

$$|a - b| = |b - a| .$$

It is important that the students realize that they always do the arithmetic inside the vertical bars first, before taking the absolute value. For example,

$$\begin{aligned} |{}^-4 - 5| &= |{}^-4 + {}^-5| \\ &= |{}^-9| \\ &= 9 . \end{aligned}$$

Class Discussion - Page 18-8a.

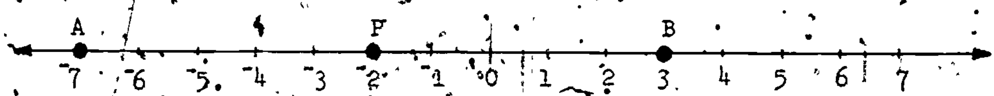
1. (a) 2 (f) 3
   (b) 2 (g) 1
   (c) 5 (h) 0
   (d) 5 (i) 4
   (e) 3 (j) 4

2. (a) 8 (f) 6
   (b) 7 (g) 4
   (c) 9 (h) 5
   (d) 2 (i) 5
   (e) 2

1. (a) 8
   (b) 10
   (c) 21
   (d) 21
   (e) 13
   (f) 13
   (g) 4
   (h) 4
   (i) 4
   (j) 4

2. (a) 4
   (b) 2
   (c) $3\frac{1}{2}$
   (d) 6
   (e) 8
   (f) 11.5
   (g) $5\frac{1}{2}$
   (h) 1.5
   (i) 2
   (j) 11.5

3. (a)



   (b) 10

Lesson 18-9.

Finding the coordinate of the midpoint of a segment on the number line is simply a matter of finding the average of two numbers. Thus, if a and b are coordinates of points on the number line then $\frac{a + b}{2}$ is the coordinate of the midpoint.

Since in the previous lesson students were working with the absolute value of a number, which is always positive, you should warn your students that we are talking now about the coordinate of a point and that the coordinate can be either positive or negative.

Class Discussion - Page 18-9.

1. (a) 5

   (b) 2

   (c) 8

   (d) $\frac{2 + 8}{2} = \frac{10}{2}$

   $= 5$

2. (a) $^-4$

   (b) $^-6$

   (c) $^-2$

   (d) $\frac{^-6 + ^-2}{2} = \frac{^-8}{2}$

   $= ^-4$

3. (a) 1

   (b) $^-6$

   (c) 8

   (d) $\frac{^-6 + 8}{2} = \frac{2}{2}$

   $= 1$

4. (a) $^-2$

   (b) 7

   (c) $\frac{^-2 + 7}{2} = \frac{5}{2}$

   $= 2\frac{1}{2}$

5. $m = \frac{a + b}{2}$

Exercises - Page 18-9b.

1. (a) $\frac{1+7}{2} = \frac{8}{2}$ or 4

(b) $\frac{^-2+4}{2} = \frac{2}{2}$ or 1.

(c) $\frac{^-8+1}{2} = \frac{^-7}{2}$ or $^-3\frac{1}{2}$

(d) $\frac{^-5+7}{2} = \frac{2}{2} = 1$

(e) $\frac{^-8+7}{2} = \frac{^-1}{2}$

2. (a) $m = \frac{0 + {}^-10}{2}$

$= \frac{^-10}{2}$

$= {}^-5$

(b) $m = \frac{^-12 + {}^-4}{2}$

$= \frac{^-16}{2}$

$= {}^-8$

(c) $m = \frac{12 + 18}{2}$

$= \frac{30}{2}$

$= 15$

(d) $m = \frac{^-1 + {}^-8}{2}$

$= \frac{^-9}{2}$

$= {}^-4\frac{1}{2}$

(e) $m = \frac{^-7 + 16}{2}$

$= \frac{^-9}{}$

$= {}^-4\frac{1}{2}$

(f) $m = \frac{^-7 + {}^-16}{2}$

$= \frac{^-23}{2}$

$= {}^-11\frac{1}{2}$

Lesson 18-10.

Ideally, we would like the student to be able to find the distance between any two points which lie on a line parallel to the X-axis by finding the absolute value of the difference of the x-coordinates, or to be able to find the distance between any two points which lie on a line parallel to the Y-axis by finding the absolute value of the difference of the y-coordinates.

In a more practical sense, it is possible that these concepts may be beyond the capabilities of your students. If this is the case, allow your students to find these distances by counting.

In the Exercises, Problems 2 and 3 do not have accompanying coordinate planes. If your students have difficulty with this problem they may plot the points, draw the segments, and then find the lengths.

Class Discussion - Page 18-10a.

1. (a) (4,7)

   (b) (4,2)

   (c) 5

   (d) Yes

   (e) Yes

   (f) 5

2. (a) $(^-2, ^-4)$

   (b) $(5, ^-4)$

   (c) 7

   (d) Yes

   (e) Yes

   (f) 7

Exercises - Page 18-10d.

1. (a) 9

   (b) 3

   (c) 9

   (d) 7

   (e) 8

   (f) 4

   (g) 6

2. $|5 - {}^-3| = |5 + 3| = |8| = 8$ or $|{}^-3 - 5| = |{}^-3 + {}^-5| = |{}^-8| = 8$

3. $|{}^-5 - 6| = |{}^-5 + {}^-6| = |{}^-11| = 11$ or $|6 - {}^-5| = |6 + 5| = |11| = 11$

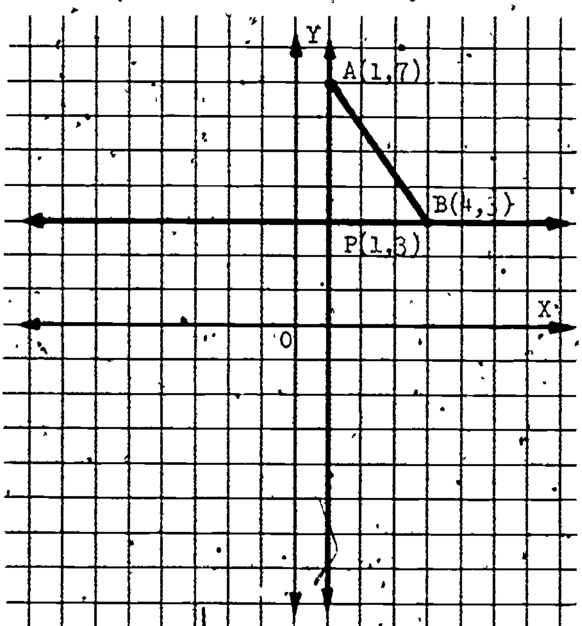
Lesson 18-11.

Again, if students are not able to handle $|x_2 - x_1|$ and $|y_2 - y_1|$, let them find the lengths of the legs of the triangles by counting.

You will probably need to review rather thoroughly the Pythagorean Property before assigning the exercises.

Class Discussion - Page 18-11.

1-7 (Construction)



3. No

6. Yes

7. (1,3)

8. a right triangle

9. (a) 4

(b) 16

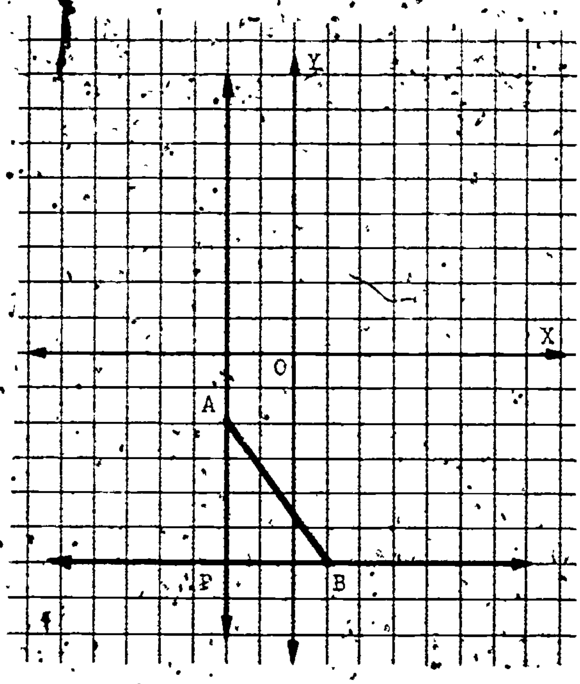
(c) 3

(d) 9

(e) 16 + 9 = 25

(f) 5

1.



Length of $\overline{AP}$ is $|{}^{-}2 - {}^{-}6| = |{}^{-}2 + 6| = |4|$ or 4

Length of $\overline{BP}$ is $|{}^{-}2 - 1| = |{}^{-}2 + {}^{-}1| = |{}^{-}3|$ or 3

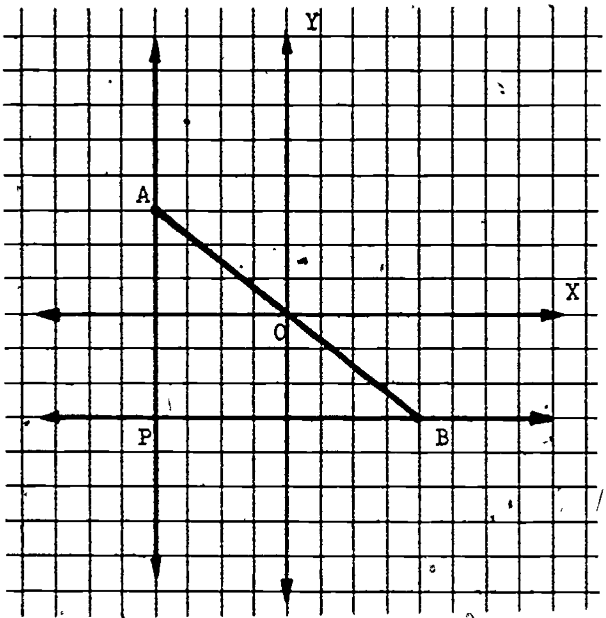
Length of $\overline{AB} = \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

2.



Length of $\overline{AP} = |3 - {}^{-}3| = |3 + 3| = |6|$ or 6

Length of $\overline{BP} = |{}^{-}4 - 4| = |{}^{-}4 + {}^{-}4| = |{}^{-}8|$ or 8

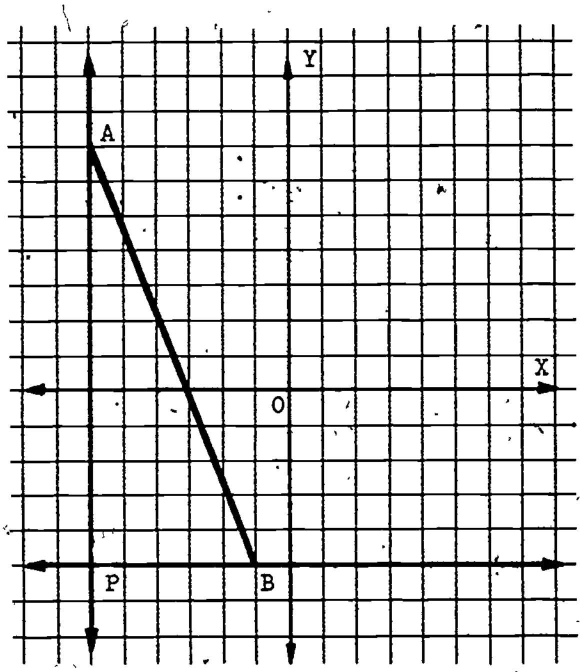
Length of $\overline{AB} = \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

3.



Length of $\overline{AP} = |7 - {}^{-}5| = |7 + 5| = |12|$ or 12

Length of $\overline{BP} = |{}^{-}6 - {}^{-}1| = |{}^{-}6 + 1| = |{}^{-}5|$ or 5

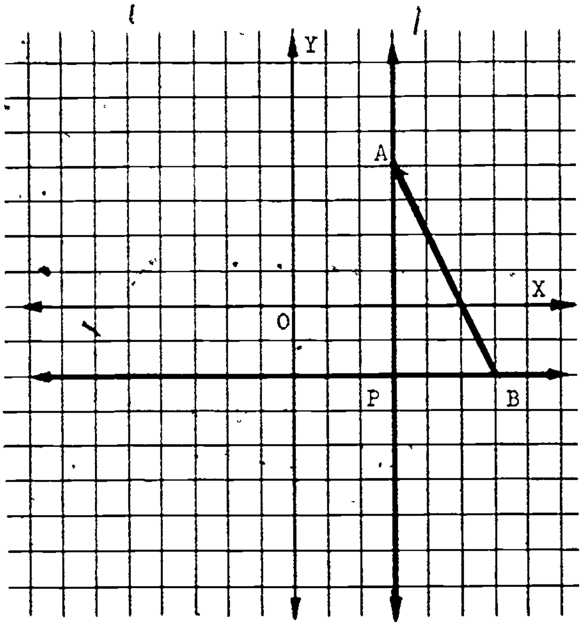
Length of $\overline{AB} = \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13$

4.



Length of $\overline{AP} = |4 - {}^{-}2| = |4 + 2| = |6|$ or 6

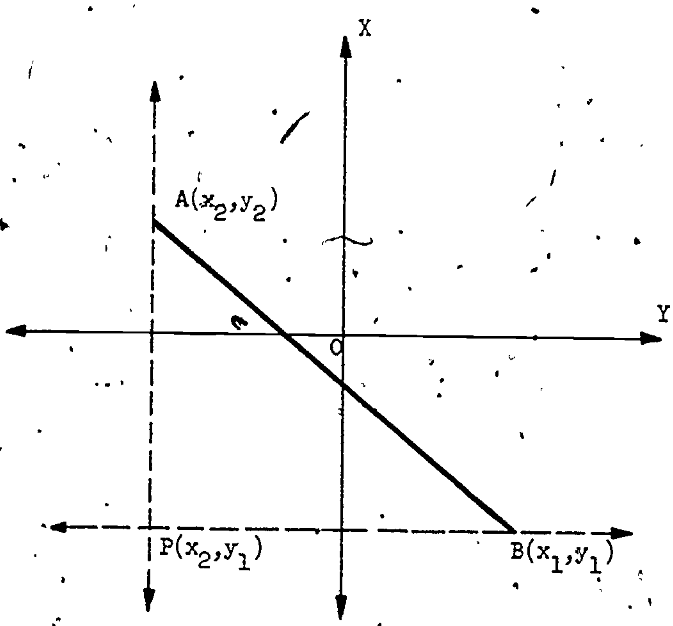
Length of $\overline{BP} = |3 - 6| = |3 + {}^{-}6| = |{}^{-}3|$ or 3

Length of $\overline{AB} = \sqrt{6^2 + 3^2}$

$= \sqrt{36 + 9}$

$= \sqrt{45}$ or $3\sqrt{5}$

5. SUPER BRAINBOOSTER.



Length of $\overline{AP} = |y_2 - y_1|$

Length of $\overline{BP} = |x_2 - x_1|$

Length of $\overline{AB} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

Pre-Test Exercises - Page 18-P-1.

1. (a) 2

   (b) 1

   (c) 2

2. (a) right

   (b) left

3. (a) $\frac{1}{2}$

   (b) $\frac{^-3}{2}$

4. (a) 4

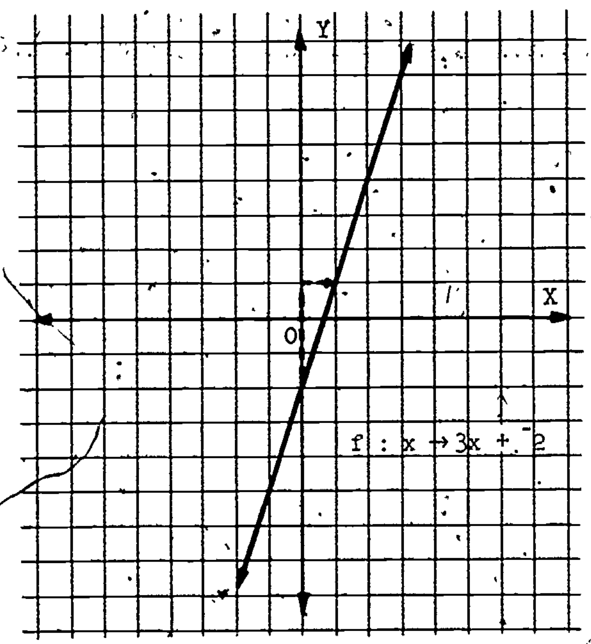
   (b) 3

5. (a) $^-3$

   (b) 5

   (c) $f : x \to {}^-3x + 5$

6.



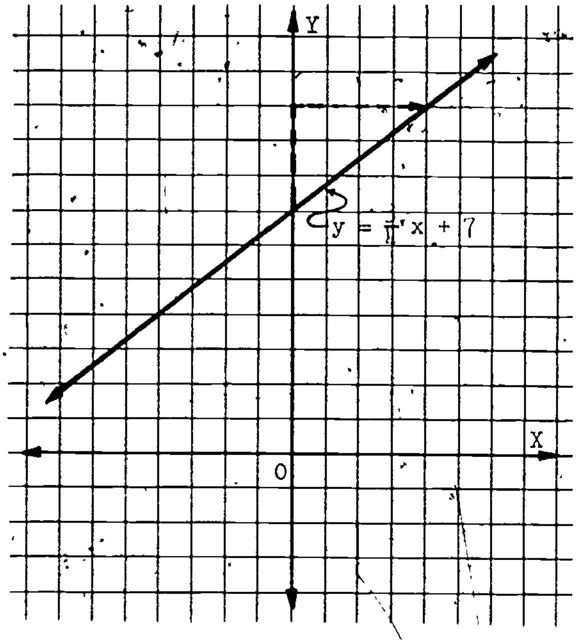
7. (a) $\frac{3}{2}$

(b) 5

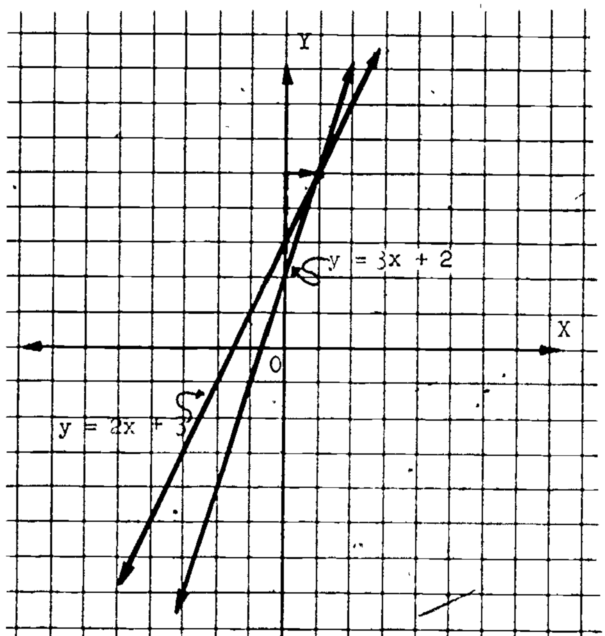
(c) $f : x \to \frac{3}{2}x + 5$

(d) $y = \frac{3}{2}x + 5$

8.



9.



The solution is <u>(1,5)</u>.

10. (a) 4

(b) $\frac{1}{4}$

(c) $\frac{^-1}{4}$

(d) $y = \frac{^-1}{4}x + 6$

11. $y = \frac{3}{4}x + 9$

12. (a) $|^-8| = \underline{8}$

(b) $|10| = \underline{10}$

(c) $|3 - 7| = |^-4|$
$= \underline{4}$

(d) $|^-5 - 10| = |^-5 + {}^-10|$
$= |^-15|$
$= \underline{15}$

13. 9

14. $\frac{1}{2}$

15. (a) Length of $\overline{AB} = |4 - {}^-5| = |4 + 5|$ or $|^-5 - 4| = |^-5 + {}^-4|$
$= |9|$ $= |^-9|$
$= \underline{9}$ $= \underline{9}$

(b) Length of $\overline{CD} = |7 - {}^-4| = |7 + 4|$ or $|^-4 - 7| = |^-4 + {}^-7|$
$= |11|$ $= |^-11|$
$= \underline{11}$ $= \underline{11}$

16. Length of $\overline{AB} = \sqrt{|2 - 5|^2 + |6 - 2|^2} = \sqrt{|^-3|^2 + |4|^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
$= \underline{5}$

Test - Page 18-T-1.

1. (a) 3

   (b) 1

   (c) 3

2. (a) positive

   (b) negative

3. (a) $\frac{1}{3}$

   (b) $\frac{^-4}{3}$

4. (a) 3

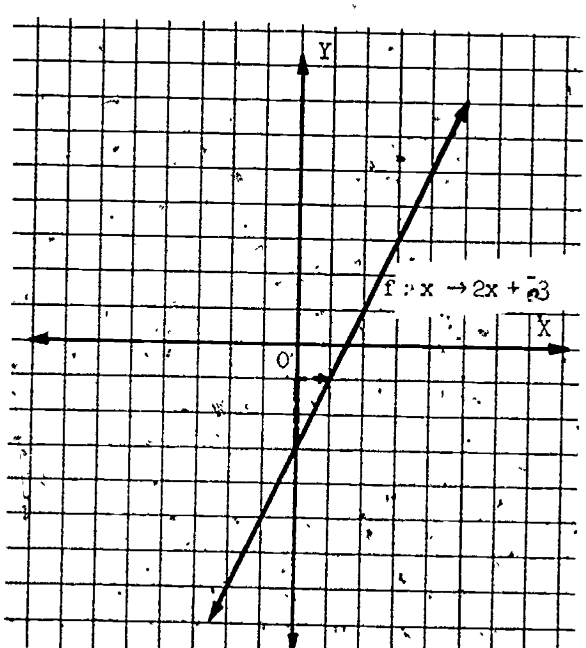
   (b) 4

5. (a) $^-2$

   (b) 3

   (c) $f : x \to {}^-2x + 3$

6.



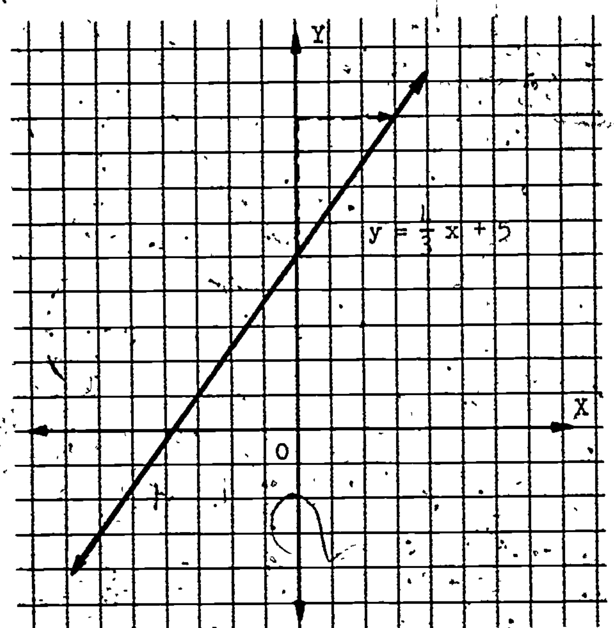
7. (a) $\frac{3}{4}$

(b) 7

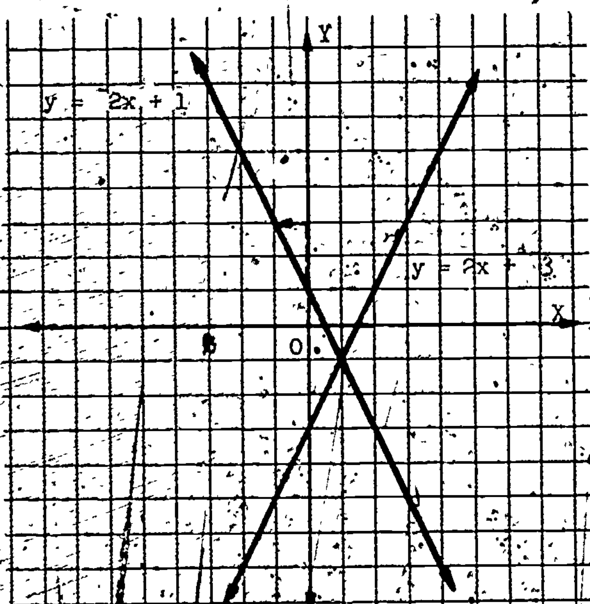
(c) $f : x \rightarrow \frac{3}{4}x + 7$

(d) $y = \frac{3}{4}x + 7$

8.



9.



The solution is $(1, ^{-}1)$

10. (a) 3

(b) $\frac{1}{3}$

(c) $\frac{{}^{-}1}{3}$

(d). $y = \frac{{}^{-}1}{3}x + 3$

11. $y = \frac{3}{2}x + {}^{-}5$

12. (a) $|{}^{-}12| = 12$

(b) $|8| = 8$

(c) $|5 - 9| = |5 + {}^{-}9|$

$= |{}^{-}4|$

$= \underline{4}$

(d) $|{}^{-}3 - 7| = |{}^{-}3 + {}^{-}7|$

$= |{}^{-}10|$

$= \underline{10}$

13. $|{}^{-}7 - 8| = |{}^{-}7 + {}^{-}8|$

$= |{}^{-}15|$

$= 15$

14. $\frac{1}{2}$

15. (a) Length of $\overline{AB}$ = $\underline{12}$.

(b) Length of $\overline{CD}$ = $\underline{9}$.

16. The length of $\overline{AB} = \sqrt{|{}^{-}9 - {}^{-}4|^2 + |5 - {}^{-}7|^2} = \sqrt{|{}^{-}9 + 4|^2 + |5 + 7|^2}$

$= \sqrt{|{}^{-}5|^2 + |12|^2}$

$= \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$